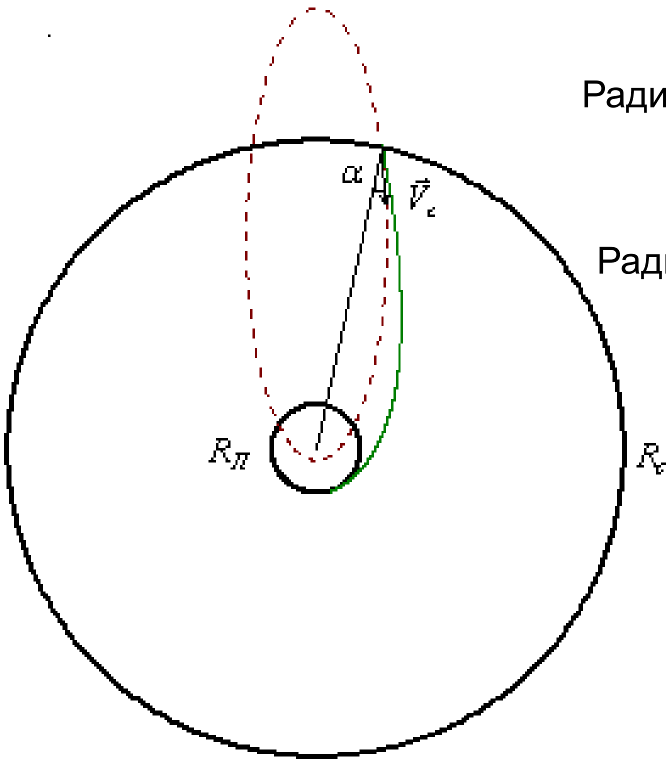
Об оптимизации посадки космического аппарата со сферы влияния Луны на её поверхность с учетом фазовых ограничений

Е.В. Заплетина, О.М. Заплетина.

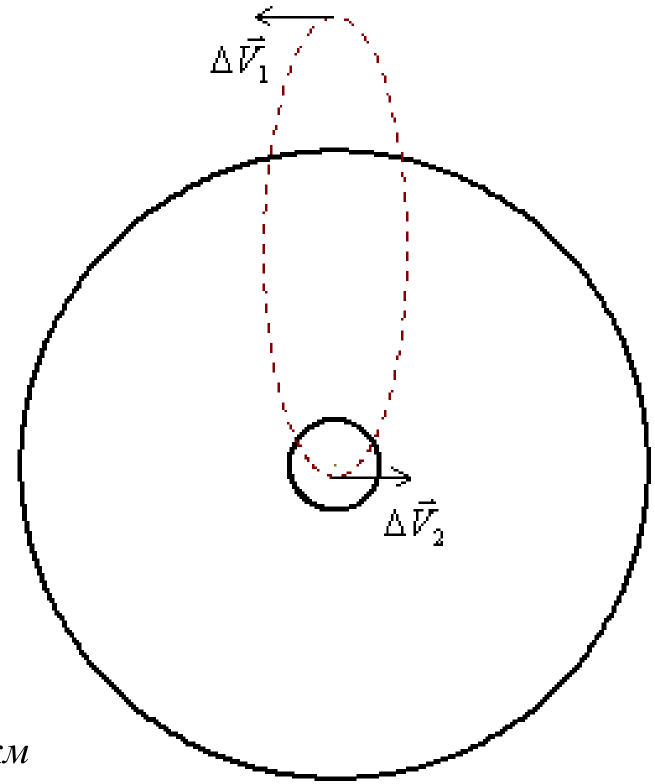
Механико-математический факультет МГУ им. М.В. Ломоносова



Радиус сферы тяготения $R_c \approx R_{\text{Л}-\text{З}}\sqrt{\frac{M_{\text{Л}}}{M_{\text{З}}}} = 42600\ \text{км}$

Радиус сферы действия $R_c \approx R_{\text{Л}-\text{З}}\left(\frac{M_{\text{Л}}}{M_{\text{З}}}\right)^{\frac{2}{5}} = 66\ \text{тыс.км}$

Радиус сферы Хилла $R_c \approx R_{\text{Л}-\text{З}}\left[\left(\frac{M_{\text{Л}}}{3M_{\text{З}}}\right)^{\frac{1}{3}} - \frac{1}{3}\left(\frac{M_{\text{Л}}}{3M_{\text{З}}}\right)^{\frac{2}{3}} - \frac{1}{9}\left(\frac{M_{\text{Л}}}{3M_{\text{З}}}\right)\right] = 58\ \text{тыс.км}$



Радиус сферы влияния $R_c \approx 1{,}15 R_{\text{Л}-\text{З}}\left(\frac{M_{\text{Л}}}{M_{\text{З}}}\right)^{\frac{1}{3}} = 102\ \text{тыс.км}$

Постановка задачи

$$J \equiv -m(T) \to \inf$$

$$\dot{\vec{R}} = \vec{V}$$

$$\dot{\vec{V}} = \frac{\vec{P}}{m} - \frac{\mu}{R^3}\vec{R}$$

$$\dot{m} = -\frac{P}{c}$$

$$(\vec{V}(0), \vec{R}(0)) = -R_c V_c \cos\alpha$$

$$R(0) = R_c$$

$$V(0) = V_c$$

$$m(0) = 1$$

$$R(T) = R_Л$$

$$V(T) = 0$$

$$0 \le P \le P_{\max}$$

Фазовые ограничения:

$$g_1 = R_Л - R(t) \le 0$$

$$g_2 = R(t) - R_c \le 0$$

$$\left|\vec{R}\right| = R, \quad \left|\vec{V}\right| = V, \quad \left|\vec{P}\right| = P$$

Необходимые условия оптимальности

$$\Lambda = \int_0^T [(\vec{p}_R, \dot{\vec{R}} - \vec{V}) + (\vec{p}_V, \dot{\vec{V}} - \frac{\vec{P}}{m} + \frac{\mu}{R^3}\vec{R}) + (p_m, \dot{m} + \frac{P}{c})]dt + \int g_1 d\nu_1 + \int g_2 d\nu_2 + \lambda_0(-m(T)) +$$

$$+\lambda_1(R(0) - R_c) + \lambda_2(V(0) - V_c) + \lambda_3((\vec{V}(0), \vec{R}(0)) + R_c V_c \cos\alpha) + \lambda_4(m(0) - 1) + \lambda_5(R(T) - R_Л) + \lambda_6 V(T)$$

$$G_1 = \frac{d^2 g_1}{dt^2}$$

$$G_2 = \frac{d^2 g_2}{dt^2}$$

$$H = (\vec{p}_R, \vec{V}) + (\vec{p}_V, \frac{\vec{P}}{m} - \frac{\mu}{R^3}\vec{R}) + (p_m, -\frac{P}{c})$$

$$\dot{\vec{p}}_R = -\frac{\partial H}{\partial \vec{R}} + \frac{\partial G_1}{\partial \vec{R}}\varphi_1(t) + \frac{\partial G_2}{\partial \vec{R}}\varphi_2(t)$$

$$\dot{\vec{p}}_V = -\frac{\partial H}{\partial \vec{V}} + \frac{\partial G_1}{\partial \vec{V}}\varphi_1(t) + \frac{\partial G_2}{\partial \vec{V}}\varphi_2(t)$$

$$\dot{p}_m = -\frac{\partial H}{\partial m} + \frac{\partial G_1}{\partial m}\varphi_1(t) + \frac{\partial G_2}{\partial m}\varphi_2(t)$$

$$\vec{p}_R(0) = \lambda_1 \frac{\vec{R}(0)}{R(0)} + \lambda_3 \vec{V}(0)$$

$$\vec{p}_V(0) = \lambda_2 \frac{\vec{V}(0)}{V(0)} + \lambda_3 \vec{R}(0)$$

$$\vec{p}_R(T) = -\lambda_5 \frac{\vec{R}(T)}{R(T)}$$

$$p_m(T) = \lambda_0$$

Необходимые условия оптимальности

$$P = \begin{cases} P_{\max}, & \chi > 0 \\ 0, & \chi < 0 \\ [0, P_{\max}], & \chi = 0 \end{cases} \qquad \vec{P} = P\frac{\vec{q}}{q},$$

$$\chi = q - \frac{p_m m}{c}, \qquad \vec{q} = \vec{p}_V + \frac{\vec{R}}{R}(\varphi_1 - \varphi_2)$$

$$H(T) = 0$$

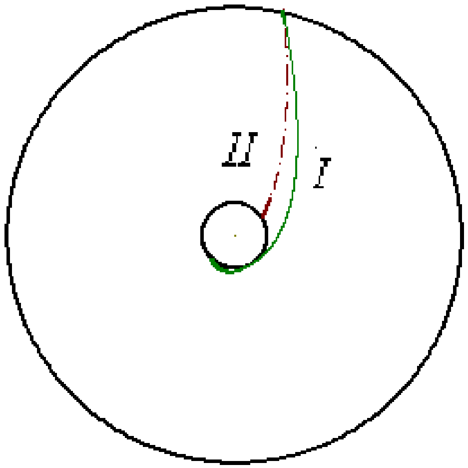
условие скачка сопряженных переменных:

$$\vec{p}_R(\tau_i + 0) = \sigma_1 \frac{\partial g_1}{\partial \vec{R}} + \sigma_2 \frac{\partial \dot{g}_1}{\partial \vec{R}} + \sigma_3 \frac{\partial g_2}{\partial \vec{R}} + \sigma_4 \frac{\partial \dot{g}_2}{\partial \vec{R}} + \vec{p}_R(\tau_i - 0)$$

$$\vec{p}_V(\tau_i + 0) = \sigma_1 \frac{\partial g_1}{\partial \vec{V}} + \sigma_2 \frac{\partial \dot{g}_1}{\partial \vec{V}} + \sigma_3 \frac{\partial g_2}{\partial \vec{V}} + \sigma_4 \frac{\partial \dot{g}_2}{\partial \vec{V}} + \vec{p}_V(\tau_i - 0)$$

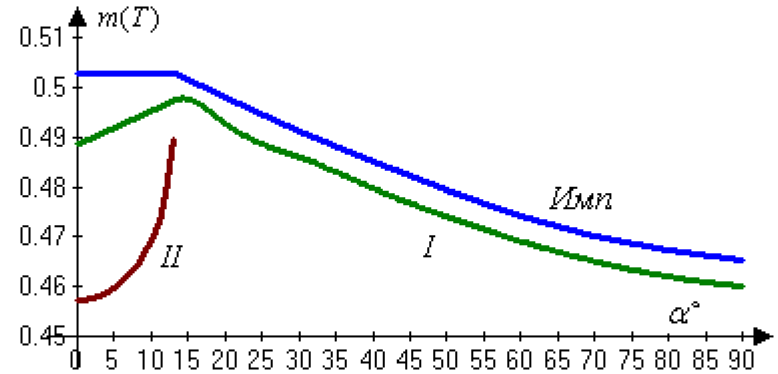
$$p_m(\tau_i + 0) = p_m(\tau_i - 0)$$

τ_i - времена выхода и схода с фазового ограничения



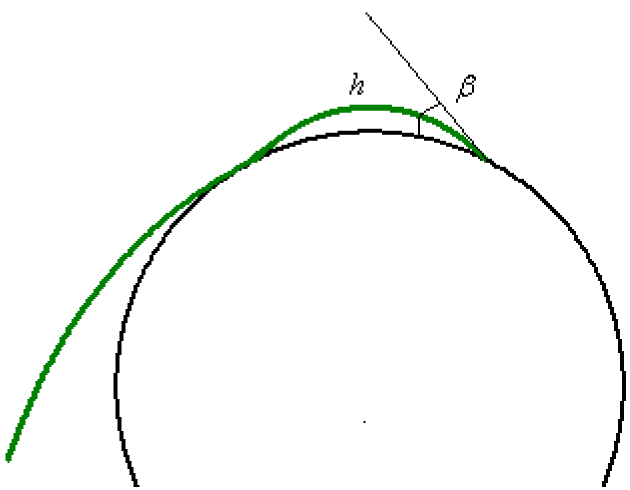
$$\alpha_{opt} = 13{,}75^{\circ}$$

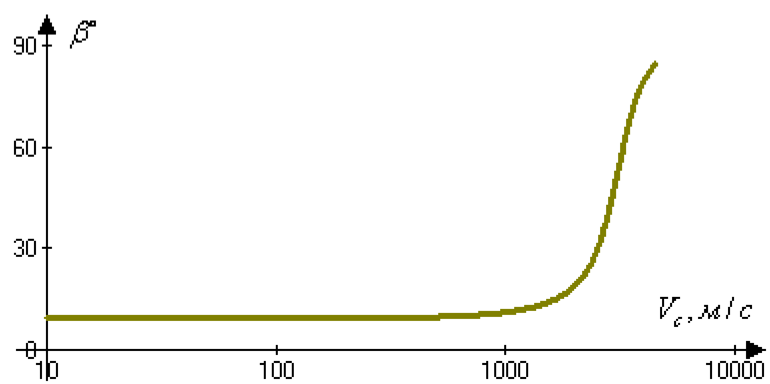
$$m_{имп} - m(T) = 5.25 \cdot 10^{-3}$$

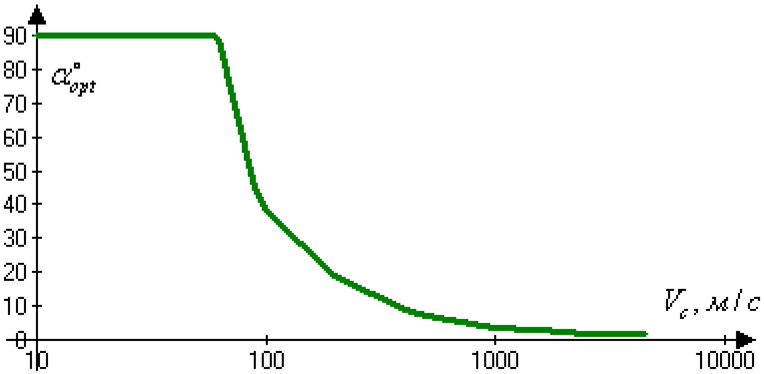


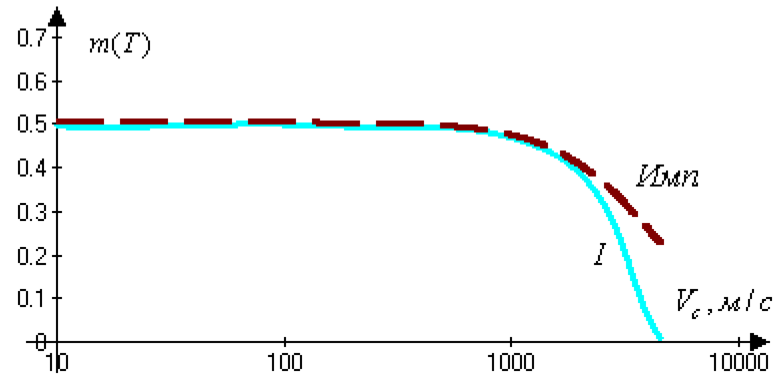
$$V_c = V_{кр}(R_c), \quad R_c = 66 \ тыс.км$$

$$P = 0{,}4 g_з, \quad P_{уд} = 350c$$



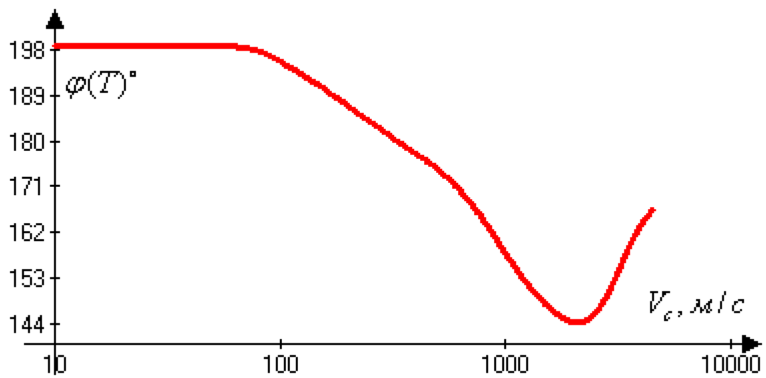


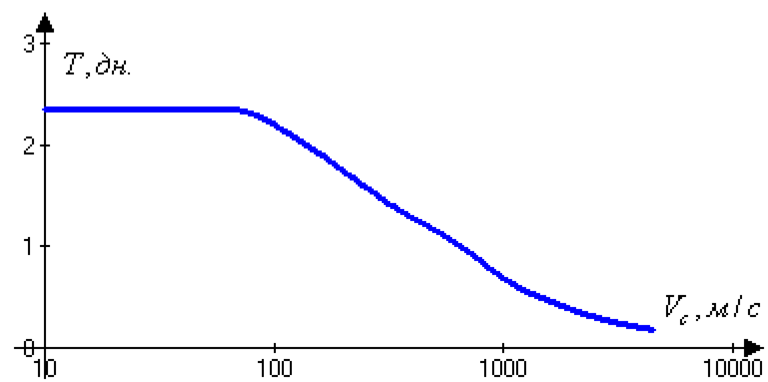


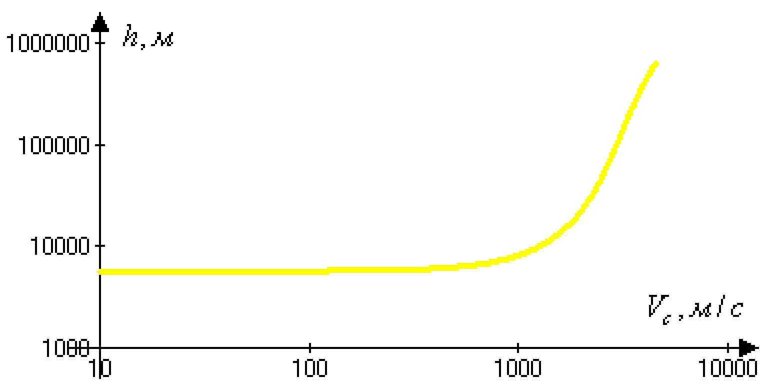


$$V_{opt} = 61{,}76 \text{ м/с}$$

$$m_{имп} - m(T) = 5.12 \cdot 10^{-3}$$







$$R_c = 66 \ \text{тыс.км}$$

$$P = 0{,}4 g_з, \qquad P_{уд} = 350 c$$