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Bipolaron Theory of Field Effect in High-Temperature Superconductors

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The field effect underlying high-temperature superconducting electronics currently has no microscopic justification. This work constructs a microscopic theory of the field effect in high-temperature superconductors, based on the translation-invariant bipolaron theory of superconductivity. It is shown that in not too strong fields the homogeneous state of the Bose condensate of such bipolarons is preserved, while its critical temperature increases. This can be used to increase the superconducting transition temperature in existing high-temperature superconductors.

**Keywords:** translational invariance, field-effect transistor, electron-phonon interaction, Bose condensate, quantum computer.
1. Introduction

Immediately after the discovery of high-temperature superconductivity (HTSC), some works appeared which predicted a significant field effect in HTSC materials [1-4].

The field effect is understood as a change in the concentration of current carriers and a change in the temperature of the superconducting transition caused by the application of an electric field to the superconductor in the normal state. Currently, this effect is actively used in superconductors with a low concentration of current carriers, which include HTSCs, to create superconducting field-effect transistors used in SC electronics [5-9].

There is currently no microscopic theory of the field effect in high-temperature superconductors due to the fact that there is no generally accepted microscopic theory of HTSC. In [10], a microscopic translation-invariant bipolaron theory of HTSC was constructed. It will be shown below that this theory can be applied to explain and calculate the field effect in HTSC.

2. Brief overview of experimental results

The field effect in substances with metal conductivity is usually small due to screening of this field by a high concentration of current carriers in them [11]. Its value in conventional superconductors does not exceed $10^{-4}$ [12].

With the discovery of HTSC, new opportunities have emerged for the study and application of this effect. Due to the unusual properties of HTSCs, which have a low concentration of current carriers, a high dielectric constant and an anomalously short coherence length, the field effect can manifest itself in them much stronger than in conventional superconductors.
Fig.1. Structure for studying the effect of a constant transverse electric field in high-temperature superconductors: $i$ – dielectric layers above and below the superconductor layer.

A typical construction for measuring the field effect involves an electrode system, a thin slab or film of dielectric, which is placed on a thin superconductor placed on a substrate (Figure 1). According to semiconductor technology, the current electrodes in Fig. 1 are called source and drain, and the dielectric electrode is called a gate. The potential applied to the gate changes the concentration of current carriers in the superconductor. The superconducting transition temperature in HTSC is extremely sensitive to even small changes in carrier concentration, which leads to a significant shift in $T_c$ under the influence of an electric field. Already in the first work [13], devoted to the study of the influence of the field on superconductivity in YBCO, such a large shift as 6 K was discovered.

Great changes in the characteristics of HTSC films under the influence of an electric field up to a change in $T_c$ by 30 K, which were observed in [14-23], led to the creation of field-effect transistors with a superconducting channel and other electronic devices based on this effect [5-9], [24-34].
3. Microscopic basis of the field effect

The proposed microscopic explanation of the field effect in HTSC is based on the idea of translationally invariant bipolarons as quasiparticles, the properties of which explain the phenomenon of high-temperature superconductivity. This may seem surprising, since an electrostatic field locally applied to a homogeneous sample violates its translational invariance. As was first shown in the author’s works, the results of which are summarized in the monograph [10], in the limit of strong electron-phonon interaction, TI bipolarons do not transform into their semiclassical analogue, and their energy lies significantly below the energy of the Pekar bipolaron (hereinafter such (bi)polarons will be called SB-(bi)polarons, that is, (bi)polarons with spontaneously broken symmetry). Thus, TI bipolarons do not “notice” the local field, which is a trap for SB bipolarons, as long as the gain in the energy of delocalized TI bipolarons exceeds the energy of the SB bipolaron captured by the trap. Being bosons, TI bipolarons at $T$ less than $T_c$ form a Bose condensate. The presence of a trap changes the properties of the Bose condensate. Its formation occurs near its energy bottom and the spatial homogeneity of the Bose condensate throughout the sample is preserved. In this case, a change in the chemical potential of TI bipolarons in the region of the applied field leads to a change in the critical value of the Bose condensate temperature in this region, that is, to a change in the temperature of the superconducting transition.

Below we will illustrate this with a particular example.

4. Theoretical calculation of $T_c$

In the absence of an external potential, the equation determining the temperature of the formation of the Bose condensate of TI bipolarons $T_c$ has the form:

$$N = \sum_k n_k, \quad n_k = \{exp(E_k - \mu)/T - 1\}^{-1}.$$  

(1)
where \( n_k \) is the Bose distribution function of TI bipolarons, \( N \) is the total number of TI bipolarons. Separating the Bose-condensate part from (1), we obtain from (1):

\[
N = N_0 + N', \quad N_0 = \{\exp(E_{bp} - \mu)/T - 1\}^{-1}, \\
N' = \{\exp(E_k - \mu)/T - 1\}^{-1},
\]

where \( N_0 \) is the number of TI bipolarons in the condensate, \( N' \) is the number of supra-condensate particles, \( \mu \) is the chemical potential such that \( \mu = E_{bp} \) at \( T < T_c \), where \( E_{bp} \) is the energy of the ground state of the TI bipolaron. The energy of the excited states of the TI bipolaron involved in (2), according to [10], has the form:

\[
E_k = [\Delta_k + E_{bp} + k^2/2M], \quad k > 0,
\]

where \( \Delta_k = \omega_k \) is the phonon frequency, which has the meaning of the superconducting gap:

\[
\Delta_k = \omega_0,
\]

- for an s-type gap, when the phonon frequency does not depend on \( k \), and:

\[
\Delta_k = \omega_0 + \Delta_0 |\cos(k_xa) - \cos(k_ya)|,
\]

- for a \( s+d \)-type gap, where \( M = 2m \), \( m \) is the effective mass of an electron (hole).

Using (2)-(4) in the case of an \( s \)-type gap, we obtain the expression for the temperature dependence of the chemical potential \( \mu \):

\[
F_{3/2}\left[(\omega_0 - \mu + E_{bp})/T\right] = c, \quad c = \left[(n^{2/3}2\pi\hbar^2)/(MT)\right]^{3/2},
\]
\[ F_{3/2}(\alpha) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{x^{1/2}dx}{e^{x}+a-1}, \quad M = 2m, \]  

(6)

where \( n = N/V \) is the concentration of TI bipolarons.

To calculate the change in \( T_c \) of a TI bipolaron gas in the presence of an external potential \( u \), we will proceed from the expression for the chemical potential of the Bose condensate of TI bipolarons, which, in the absence of an external potential, is determined by relations (6).

We will assume that the volume to which the external potential is applied is small compared to the volume of the entire SC. Due to the spatial homogeneity of the distribution of TI bipolarons when \( u \) in absolute value is less than the critical value, their concentration in the region of the applied field is the same as in the volume of the entire sample.

Fig.2 Shown are the temperature dependences of the chemical potential \( \mu \) on the segments \([T_{c1}, 1.31T_{c1}]\) for \( T_{ci}, i=1, 2, 3 \) and \([T_{c4}, 1.31T_{c4}]\) for \( i=4, 5, 6 \), corresponding to various values of the phonon frequency \( \omega_i = \omega^* \tilde{\omega}_i \), \( \omega^* = 0.005 \) eV; \( \tilde{\mu} = (\mu - E_{bp})/\omega^* \), \( \tilde{u} = u/\omega^* \), \( \tilde{T} = T/\omega^* \). The dependencies are constructed for the values: \( \tilde{T}_{c1} = 27.3 \tilde{T}_{c2} = 30; \tilde{T}_{c3} = 32; \tilde{T}_{c4} = 42; \tilde{T}_{c5} = 46.2; \tilde{T}_{c6} = 50; \tilde{\omega}_1 = 0.2; \tilde{\omega}_2 = 1; \tilde{\omega}_3 = 2; \tilde{\omega}_4 = 10; \tilde{\omega}_5 = 15; \tilde{\omega}_6 = 20; \tilde{u} = -2. \)
Figure 2 shows the temperature dependences $\mu(T)$ for various values of the parameter $\omega_i = \bar{\omega}\omega^*$, where $\omega_i$ is the frequency of optical phonons, $\omega^* = 0.005$ eV.

In the simplest case of a constant external potential $u$, in a superconductor in the gate region, as shown in Fig. 1, in relations (1) in the field region it is sufficient to replace $E_{bp}$ by $E_{bp} + u$. This leads to an increase in the temperature of formation of the Bose condensate, which is determined by the points of intersection of the line $u = \text{const}$ with the curves $\mu(T)$. The figure suggests, for example, that at the well depth $|u|=0.01\text{eV}$, the increase in the temperature of the Bose condensate will be $T_c(|u|=0.01\text{eV})/T_c(0)=1.17$ for $\bar{\omega}_i = 0.2$ (which corresponds to $\omega_i = 1 \text{meV}$). As the potential well depth and phonon frequency increase, the SC transition temperature also increases. However, it should be borne in mind that at large $\omega_i$ the strong coupling approximation for TI bipolarons becomes inapplicable, and at large well depths the condition of translational invariance fails. Note that the external potential on the control electrode, due to the presence of a double layer and the corresponding potential jump at the superconductor-insulator interface, can be hundreds of times higher than $u$ by their absolute values.

In the case of an $s+d$ gap of type (5), the first term in the right-hand side of (5) corresponds to the contribution of the $s$-type wave, and the second term – to the contribution of the $d$-type wave. Expression (5) yields the condition, when the main contribution to integral (2) comes from $k \approx \sqrt{2MT}$ and $ka \ll 1$. In this case, from (2) and (5) we obtain: $\Delta_0|\cos k_xa - \cos k_ya|/T \equiv \Delta_0Ma^2/\hbar^2 \ll 1$. Thus, for $\omega_0 \geq \Delta_0Ma^2T/\hbar^2$, the main contribution to the integral will be made by the $s$-type wave. In this case the results obtained above for the case of $s$-type condensate will remain unchanged. For example, in the case of YBCO the value of $\omega_0/\Delta_0$ is $\approx 0.15$ [35], [36], and the $s$-approximation condition in this case is satisfied with high accuracy.

The results obtained in the region of their applicability are quite general and, in particular, do not depend on the shape of the region to which the external field is
applied. When the sign of $u$ changes to the opposite (repulsive potential), TI bipolaron superconductivity (for finite values of $u$) near $T_c$ ceases to exist. Thus, near $T_c$, by simply changing the sign of the applied potential, one can transfer the sample from the SC state to the normal state and back. This effect currently underlies the operation of superconducting field-effect transistors. Note that at temperatures lower than $\text{min}[T_c(-u), T_c(u)]$, the current-voltage characteristic will remain unchanged when the sign of the gate potential changes. If the experiment is carried out near $T_c$, then an increase in the critical temperature in the region of the control electrode will not affect the value of the critical current $J_c$. On the contrary, a decrease in $T_c$ and, as a consequence, the transition of the region where the control electrode occurs to a resistive state will lead to a decrease in $J_c$. Thus, in the experiment, regardless of the sign of the gate voltage, the value of $J_c$ will either remain unchanged or decrease.

5. Possible applications

As noted above, the first phenomenological theories of the field effect for the case of HTSC [1-4], as well as subsequent ones [37-40], used an analogy with the ordinary field effect in semiconductors. Based on the BCS theory, these works can hardly explain the field effect in HTSC films, in which the smallness of the coherence length plays a significant role. In [41], the ideas of the field effect were used to explain the phenomenon, similar to how this is done in quantum electrodynamics.

This work uses a microscopic approach based on the results of the translation-invariant theory of superconductivity [10]. According to the microscopic theory presented, an external electrostatic field disrupts the homogeneity of only ordinary (unpaired) current carriers, changing their concentration in the field region and leaving the concentration of TI bipolarons constant throughout the sample. This, in particular, explains why islands of superconductivity with increased $T_c$ arise in dirty
superconductors, which (the temperature) persists even when the entire sample outside the region of the applied field goes into the normal state.

The theory developed gives a value for the shift of $T_c$ in the field that is consistent with experiment and can be used to increase the critical temperature of the superconducting transition of known superconductors. As noted in [42], using the field effect by changing the concentration of current carriers, it is possible to create a SC state even in materials that are not SC under normal conditions, or in crystals in which chemical doping with impurities supplying current carriers is impossible. An example is a compound KTaO$_3$, in which a SC with $T_c=0.3$K was induced using the field effect [43].

Modulation of HTSC parameters by an electric field, that is, switching the SC film from the normal state and back, is used in microelectronic devices such as transistors, switching elements-electric cryotrons, control filters, mixers, delay lines and others. These devices, in turn, pave the way for a large number of new tunable gate devices, SQUIDS, magnetometers, radiation detectors, and quantum computers [7], [25], [27-28], [44, 29, 30, 31, 33].

References


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