Improved model of IBL sunlight simulation

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Abstract

This paper considers the use of high dynamic range images as a source of natural daylight illumination in virtual scenes containing objects with complex optical properties (such as automotive multilayer paints). Physically justified algorithms for calculating the luminance of an object point and recognition of the sun to replace it by separate light source are outlined. An analysis of the correctness of the available high dynamic range images is performed, and their drawbacks are discussed. Two algorithms that allow to compensate the drawbacks of the representation of the visible sun in the high dynamic range images and thus perform a more accurate illumination simulation are proposed and elaborated.. The correction of sun representation is based on the sky models adopted by the International Commission on Illumination (CIE). The modified illumination simulation algorithms make it possible to produce more accurate images. The algorithms are semi- automatic and robust with respect to errors in high dynamic range images. These algorithms are used in our computer graphics system for generating realistic images.

CR Categories: I.3.3 [Computer Graphics]: Picture/image generation – Rendering algorithms; I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism – Lighting, sunlight, shading.

Keywords: HDRI, IBL, sunlight, Monte-Carlo ray tracing, BRDF, physically accurate rendering

1 Introduction

A large dynamic range of luminance, when the ratio of the brightest region in the field of view to the dimmest region can be millions, is easily perceived by human eyes. People usually can see objects both under the lunar light (when the illuminance of a horizontal surface is about 0.2 lux) and under the direct solar light (when the illuminance of a horizontal surface is hundreds of thousands of lux). The sensitivity of the human vision system lies within the range of 10^{-6} to 10^{8} cd/m^2. However wide range of intensities cannot be represented on conventional display devices because they have a low dynamic range (usually - several hundreds). Modern development in the field of computer graphics increasingly focuses on high dynamic range images (HDRI). In an HDR image, the value of each pixel is represented by a real physical value. Floating point values make it possible to represent real-life intensities of light even beyond the sensitivity of the human eye. HDRI and the conventional format for their representation were proposed in [Ward 1991] (the standard extension for the RGBE files is hdr).

Initially HDRIs were produced exclusively by physically based lighting simulation systems. But now HDRIs of real scenes can be produced by processing a sequence of photographs of the same scene made with different exposures or using a panoramic video scan of the scene [Reinhard et al. 2006]. It is the capability to obtain realistic images of objects and landscapes retaining the brightness and color of real world that opens rich possibilities for using HDRIs in computer graphics. In particular for the technology called Image-Based Lighting (IBL) [Debevec 2002].

Most of algorithms used to generate realistic images is based on simulating the light propagation in various media. This requires that light sources are specified. In the IBL technology, an image is used as a light source. For example, one can use a photo of the sky or a photo of a show-room as a light source and simulate the illumination of objects under these conditions. The specification of lighting conditions by other means (for example, by specifying a set of appropriately located light sources) can be a very costly process. The HDRI used in image-based lighting must have two properties. First, it must contain points for all the directions of the environment being photographed; i.e. it must be a spherical panoramic image. Second, the value of each pixel must be proportional to the amount of light that comes from the given direction in the real world.

Many lighting simulation and visualization systems, such as 3dMax (Mental ray, VRay), Softimage, Maya, and others, use HDRIs as background panoramic images which significantly improves the quality of the resulting images. Some of the systems calculate the scene illumination using HDRI as a light source. In many cases the algorithms and methods used for the illumination calculation are well suited only for the case of lengthy and area light sources such as the cloudy sky. Such light sources yield fuzzy soft shadows with an implicit direction to the light source.

LightWave 3D is an example of a realistic visualization software that uses HDRIs both as background images and for calculation of the scene illumination. Global illumination of a scene is calculated in LightWave 3D by the radiosity method. This method works well when the light source is rather uniform which is the case when there is used an HDRI without the sun; then high-quality soft shadows are obtained. However if the HDRI contains a powerful light source, such as the sun, the system cannot automatically produce sharp shadows. To solve this problem LightWave 3D users are supposed to manually install an additional light source in the direction of the sun with the corresponding scaling (reduction) of the values in the HDRI. The same technique — an additional light source which should be specified manually — is used in some other visualization systems.

The companies engaged in the commercial production of HDRI libraries try to keep the sun at the same position in the entire series of images. When various scenes are composed and the properties of materials are specified, this ensures the

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interchangeability of the HDR panoramic images without the need for additional manual adjustment of the sun position.

In this paper we describe an image-based lighting technique developed by the authors that reconstructs correct lighting for the HDRIs containing such a powerful light source as the sun. An analysis of some available HDRIs obtained by the multiple photography method with various exposure times is performed. Modifications of the illumination calculation and realistic image generation techniques are proposed. The proposed modifications take into account the drawbacks of the available HDRIs.

The reconstruction of high dynamic light source representation is similar to LDR2HDR problem related to reproduction of low dynamic range (LDR) images on recent HDR displays. There are several published solutions like the improvement of highlight appearance [Meylan et al. 2006], elaboration of an inverse tone mapping operator [Banterle et al. 2007] or enhancement of dynamic range for video [Rempel et al. 2007]. However in our case, instead of improving the appearance of a picture on an HDR display, we needed to use the results of the correct sun light reconstruction in further simulations. So the mentioned LDR2HDR algorithms are not applicable for us.

2 Calculation of illumination

Our project was devoted to physically accurate rendering of car models covered by pearlescent paints used in automotive industry. Such paints can have quite complex appearance. In general case the painted surface can be described by a bi-directional reflection distribution function (BRDF). BRDFs that we use come from real-world data. They are obtained by either measuring the BRDF samples (the appropriate measurement setup was elaborated and works in our institute [Letunov et al. 2006]) or by deriving BRDF from the microstructure of the materials [Ershov et al. 2004]. BRDF obtained in such ways exhibits complex structure which is hard to approximate by a particular BRDF model. Therefore, the only way to represent BRDFs which suit to our purposes is tabular representation. Usually the paint BRDF varies more rapidly near the direction of specular reflection, so the following parameterization of BRDF is used. The BRDF (isotropic or 3-dimentional for paint representation) coordinates are: σ - angle between direction of incoming light and surface normal, ϑ - angle between direction of specular reflection and direction to observer, φ - angle of rotation of the observer direction around the direction of specular reflection (fig. 1).

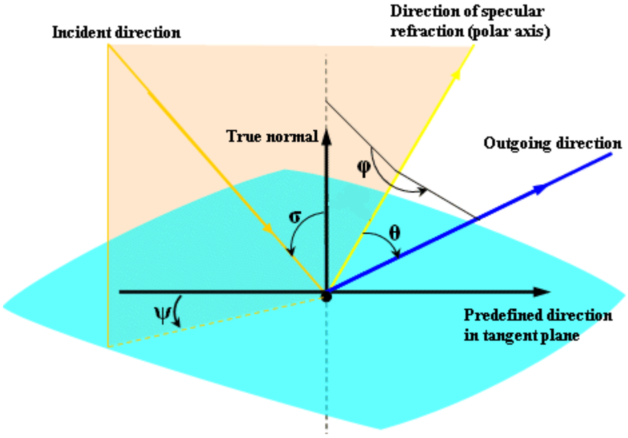


Fig. 1. BRDF coordinate system.

Such coordinate system allows to specify BRDF values for any direction and adaptively ensures more dense mesh of values near the specular direction.

The luminance of a point of a surface whose properties are described by a BRDF can be calculated in the following way:

$$L = \frac{1}{\pi}\int_{(\vec{n}\cdot\vec{\omega})\geq 0} \mathrm{BRDF}(\sigma(\vec{n},\vec{\omega},\vec{v}),\vartheta(\vec{n},\vec{\omega},\vec{v}),\varphi(\vec{n},\vec{\omega},\vec{v}))(\vec{n}\cdot\vec{\omega})I(\vec{\omega})d^2\omega,$$

where $\vec{\omega}$ – the illumination direction;
$\vec{v}$ – the observation direction;
$\vec{n}$ – the surface normal at the point;
I – the illumination intensity;
$I(\vec{\omega})d^2\omega$ – the intensity of the illumination created by all HDRI points belonging to the solid angle $d^2\omega$;
σ, ϑ, φ – angles defined above.

Each lighting direction $\vec{\omega}$ illuminates the surface from one side only. The backside of the surface is in shadow with respect to the given direction. This is taken into account in the integration by the condition $(\vec{n}\cdot\vec{\omega})\geq 0$.

The backward Monte Carlo ray tracing technique can be considered as an effective method for calculation of this integral for almost any kind of BRDF. But this is true in case the illuminating HDRI is relatively uniform; i.e. this method is best suited for a uniform lighting created by a cloudy sky. However if the HDRI contains a bright light source (for example, the directly visible sun), the computational effort is significant. Separation of bright light sources from the rest of HDRI makes it possible to solve this problem. Panoramic HDRIs are supposed to be located on the infinite sphere; therefore it is reasonable to replace the distinguishable bright regions by parallel light sources.

As we had to simulate not only the exterior but also the interior of a car, the calculation of global illumination is a must for our project. To calculate the global illumination the light reflected from other diffusive surfaces of the scene must be taken into account. We calculated this component with the help of the forward Monte Carlo ray tracing. However the direct application of this method is inefficient in the case of an HDRI used as a light source. For this reason, when calculating the global illumination, the light sources (i.e. regions with high intensities) are selected in the spherical panoramic HDRI and replaced by the corresponding parallel sources. The calculation of the global illumination component is performed for these more conventional light sources.

Selection of bright regions and their replacement by parallel light sources also solves the problem of generating sharp shadows due to the sun or other bright light sources. Most of the available software packages can not calculate sharp shadows from the sun recorded in HDRI automatically. They prompt the user to manually install an auxiliary light source that produces sharp shadows. The recognition and selection of light sources enabled us to correctly simulate the illumination for HDRIs with the visible sun without using artificial light sources.

To distinguish the light sources, we use three parameters that control the recognition of bright regions and the calculation of the source power. The first parameter is used to determine the direction to the light source and specifies the intensity threshold of an HDRI point. It is used to distinguish the points that can be considered as separate light sources. The second and the third parameters are used to determine the entire bright regions and, finally, to calculate the luminance of the light source. The second parameter determines the maximum radius of a bright region. If a point is sufficiently bright (i.e. if its luminance exceeds the threshold specified by first parameter) and is far from the initial bright point more than the second parameter, then it is regarded as another light source. The third parameter specifies the minimal luminance of an HDRI point that is regarded as a part of the light source. Thus, a bright region is defined as the set of points that have the luminance greater than the minimal brightness and are not farther away from the initial brightest point for a distance defined by the second parameter.

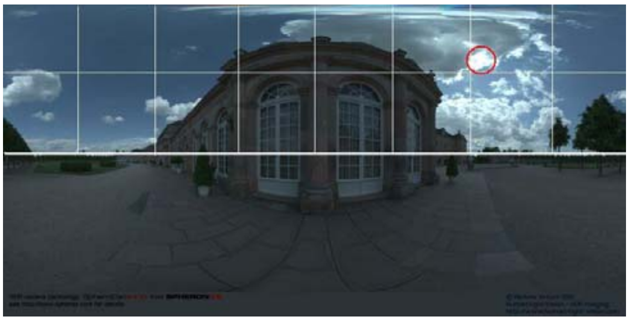


Fig. 2. Recognition of bright light sources.

The values of the HDRI pixels that belong to the selected light sources are nullified. As a result, for the further calculations, we obtain a sufficiently smooth illumination function defined by the smoothed HDRI and a set of bright parallel light sources. The bright regions are considered only for the upper hemisphere as is shown on fig. 2. The exact values of the first and third parameters are calculated using an empirical formula. They depend on the average intensity of the segment of the image (to this end the HDRI is subdivided into segments corresponding to the pyramid of vision $45^0 \times 45^0$ as it is illustrated in fig. 2). The optimal value of the maximum radius of the bright regions is also determined empirically.

3 Analysis of HDRI

Despite the powerful mechanism of using an HDRI as a light source for simulating illumination described above its use does not guarantee that realistic images of photographic quality shall be obtained. An analysis of several dozens of HDRIs obtained from various sources showed that the main problems are related to the absence of sharp shadows and correct illumination for HDRIs containing the visible sun. These factors lead to less realistic resulting images.

The developed algorithms for constructing images illuminated by HDRIs are physically justified and we expected that they would generate correct images. However, this was the case only when the HDRI used as a light source was physically correct. For this reason, we made an attempt to analyze some available HDRIs. There are two most important characteristics of HDRIs:

- the dynamic range,
- the correctness of the sun representation.

All the HDRIs we analyzed were produced using the devices manufactured by SpheronVR [Spheron] both by this company itself and by Dosch Design [Dosch].

To generate a correct IBL image, the input data (the HDRI panorama) must represent the conditions under which it was taken as accurate as possible. Therefore if the actual range of luminances, for example, the ratio of the sun luminance to the luminance of a black object indoors, is large, then the range of luminances in the HDRI must be of the same order of magnitude. As the major difficulties occur in the HDRIs with the visible sun we analyzed such type of HDRI. The analysis of such images was based on the standard clear sky model and the partly cloudy sky model elaborated and approved by the Illuminating Engineering Society of North America (IESNA) and the CIE [Rea 1993]. For the clear sky model the ratio of the maximum luminance of the sunny sky to the minimal luminance is of order of 23737; therefore the dynamic range of the sky must have the order of 10^4. For the partly cloudy sky model the ratio of the maximum luminance of the sunny sky to the minimal luminance is of order of 4722; therefore the dynamic range of the sky must have the order of 10^3. Finding all the points in an HDRI that belong to the sky is generally a difficult problem. It is much easier to calculate the total dynamic range of the HDRI that includes the points in the lower hemisphere of the panorama containing the images of the soil, buildings, etc. We investigated tens of HDRIs produced by Spheron. For most of them the lower hemisphere adds two orders of magnitude to the dynamic range. Thus, an approximate estimate of the dynamic range of an HDRI with the directly visible sun is at least 10^6, and an estimate for the partly cloudy sky is 10^5. If the dynamic range of an HDRI is less, then we can conclude that this image does not give an accurate representation of the reality. And this will result in incorrect simulation results.

While analyzing HDRIs we solved the following problem. The lower bound of the luminance can contain very small nonzero values (for example, 10^{-15}, 10^{-8}, or 10^{-3}). They are perceived as very dark regions not distinguishable by the human eye. Actually they can be just noise generated by the digital camera. However they produce very high dynamic range. To be able to discard such small values all luminance values must be transformed to the standard nit (or cd/m^2) units. In particular the representation of the luminance in nit units is used to measure the luminance of computer displays. It is known that the maximal luminance of CRT displays is about 250–300 nit. Therefore we can replace the values that are less than 1 nit by unity because these values cannot be distinguished by the human eye and can be considered as noise.

The analysis of the sky HDRIs produced by Dosch shows that none of HDRI (of ten available for us) has acceptable dynamic range. The maximal dynamic range value is only 3961, i.e. $4.0 \cdot 10^3$ only. This is by two or three orders of magnitude less than what was expected. Some of these images have the range of only 10^2 which corresponds to ordinary photographs represented in the RGBE format. The simulation of illuminance based on such an image cannot be correct and the resulting images will inevitably have the drawbacks such as soft shadows from the sun and insufficient illumination of scene objects.

The analysis of the sky HDRIs produced by Spheron demonstrated much wider range of these images. The two images have an acceptable range (about $6 \cdot 10^5$). The range of another two images is also close to an acceptable one ($\sim 6 \cdot 10^4$). Figure 3 shows the result of generating a realistic image of a car illuminating by the HDRI “SpheronVR direct sun roof 003.hdr”. The shadows from the sun are quite sharp. The appearance of the car’s paint proves that it is illuminated correctly.

However, when looking at fig. 3, it seems that the sun is insufficiently bright. This is seen from the shadow, which is brighter than can be expected. The flare from the sun on the car's body is blurred, which is a characteristic of weak light sources.

Fig. 3. A car model illuminated by HDRI with acceptable dynamic range.

To investigate the problem the correctness of the sun representation was analyzed. As in the previous case the CIE and IESNA standards were used to verify it. The illuminance of a horizontal surface for the clear sky model is formed by the illuminance created by the sun and the illuminance created by the sky with the sun excluded. According to the CIE standard the ratio of the illuminances produced by these elements is about 4 : 1; that is, the sun illuminates the horizontal surface four times better than the other part of the sky. We calculated the ratio of the illuminances of the horizontal surface produced by the sun and by the sky without the sun for two «the best» Spheron's HDRIs: «SpheronVR direct sun roof 003.hdr» and «a52_Lagerschuppen.hdr». For the first HDRI this ratio is 1 : 1; i.e. the illuminance produced by the sun is equal to the illuminance produced by the sky. For the second HDRI this ratio is 1 : 5; i.e. the illuminance produced by the sun is five times less than the illuminance produced by the sky. Both images represent a clear sky with the visible high standing sun. Therefore the representation of the sun in these HDRIs is incorrect. The luminance of the sun is strongly underestimated.

More detailed description of result of the HDRI analysis was published in [Voloboy et al. 2006].

4 HDRI drawback compensation algorithms

After analysis of the available HDRIs we realized that the algorithms designed for lighting simulation and generation of realistic images must be modified to compensate for the drawbacks of these images. The most important thing is compensation of the drawbacks related to the representation of the sun. It would be incorrect to change the luminance of the sky because this is the sun luminance that is underestimated. The manufactures of devices for producing HDRIs claim that the luminance of all the objects except for the sun is measured with a high accuracy.

As we see the main problem is the underestimation of the sun luminance which is valid even for the images having an acceptable dynamic range. This problem can be solved using the CIE standard sky models. The first straightforward approach is to calculate the illuminance of the horizontal surface from the sun and from the modified upper hemisphere of the panoramic HDRI (the sky with the sun excluded). Then, the luminance of the light source is scaled up so that the illuminance produced by it is four times greater than the illuminance produced by the remaining part of the sky. The automatic detection of HDRIs with the purpose to determine when the light sources must be scaled up is a severe problem. Based on an automatic analysis it is difficult to differentiate between the sun and an artificial bright light source. A natural decision in this case is to enable the user to set the scaling up mode for the insufficient luminance of the sun depending on a visual estimate of the HDRI.

Fig. 4 shows the result produced by our compensation algorithm. The fig. 4a shows the image of a car model placed in the Spheron's HDRI «a52_Lagerschuppen.hdr» with the visible sun (with the dynamic range of $5.6 \cdot 10^4$, the ratio of the illuminance of the horizontal plane from the sun to the illuminance from the sky is 1 : 5; i.e. the sky is five times brighter than the sun.

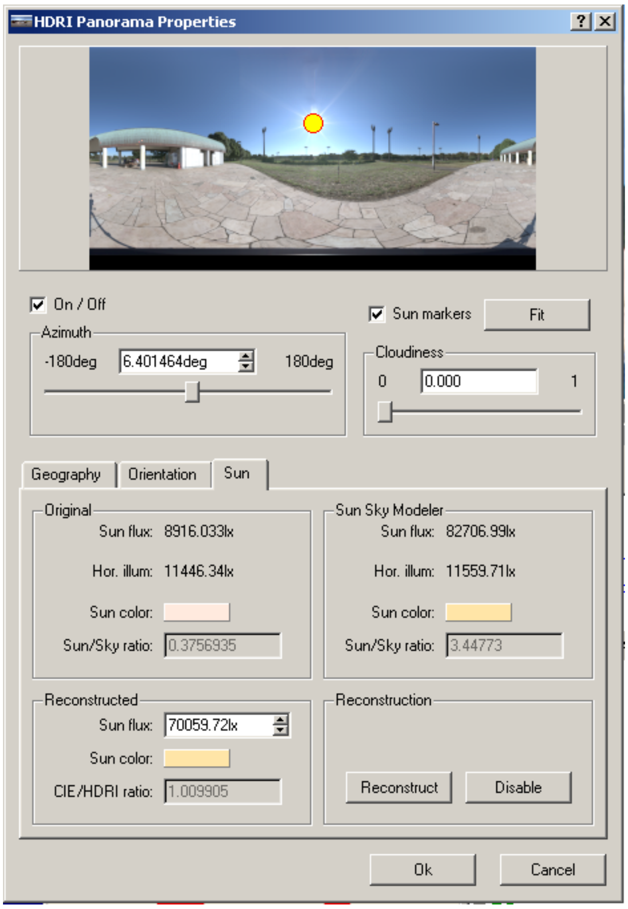
Fig. 4a. The image generated without the HDRI drawback compensation algorithm.

Fig. 4b. The image generated with the HDRI drawback compensation algorithm applied.

It is seen that the car is insufficiently illuminated for a bright sunny day: the shadows from the sun look pale. The fig. 4b shows the image obtained using a compensation of the incorrectness of the sun representation. The illuminance of the car is acceptable. A sharp shadow corresponding to a bright sunny day is visible under the car.

While the compensation algorithm is generally automatic, the recognition of HDRI containing sun cannot be done automatically (taking into account that sun can be represented incorrectly). Therefore a minimal user interface is necessary here: user will specify whether to apply the sun HDRI compensation algorithm or not.

Fig. 5. Sun position fitting procedure.



☑ On / Off

☑ Sun markers

Azimuth

-180deg 6.401464deg 180deg

Cloudiness

0 0.000 1

Original

Sun flux: 8916.033lx

Hor. illum: 11446.34lx

Sun color:

Sun/Sky ratio: 0.3756935

Sun Sky Modeler

Sun flux: 82706.99lx

Hor. illum: 11559.71lx

Sun color:

Sun/Sky ratio: 3.44773

Reconstructed

Sun flux: 70059.72lx

Sun color:

CIE/HDRI ratio: 1.009905

Reconstruction

Fig. 6. UI panel of sun reconstruction.

However in many HDRIs the calculation of illumination from whole sky semi-sphere is impossible because the sky is shielded by trees or buildings. For these HDRIs application of the drawback compensation algorithm is problematic. Therefore another approach was implemented. In this second approach the total flux of the parallel light source representing the sun is reconstructed from the CIE sky model.

To specify the sky model parameters a user should set geographical location (by specifying longitude and latitude or by choosing a predefined location, for example, Washington DC, USA), date and time which correspond to the HDRI. Also user should specify HDRI orientation (azimuth direction). The time and HDRI orientation can be calculated automatically by fitting algorithm which is illustrated on fig. 5. The red circle there corresponds to the sun area identified by analysis of HDRI. The yellow circle on the figure corresponds to the sun position calculated from the CIE sky model. The fitting procedure matches the sun positions varying time and HDRI azimuth parameters. After the sky model parameters are set, a user should pick any point belonging to open sky (as we cannot automatically recognize sky points with certainty). This way the correct values of whole sky including sun will be reconstructed for given HDRI. Fig. 6 shows the final value of reconstructed sun total flux (70059.72 lx) which is significantly different from original sun total flux (8916.03 lx) calculated from HDRI.

Fig. 7 shows the result produced by our sun reconstruction algorithm. Fig. 7a shows the image of a car model placed in the HDRI with the visible sun whose total flux was calculated from HDRI. The flux is equal to 8916.03 lux. It can be seen that sun illumination does not correspond to bright sunny day. Fig. 7b shows the image obtained after sun reconstruction (total flux of the sun is 70059.72 lux). On fig. 7b the illumination of the car is quite acceptable; the sharp shadows look realistic.

5 Results

The described algorithms were elaborated and implemented for the realistic visualization system Inspirer2 (former Fly) [Ignatenko et al. 2004]. Figures 3-7 were generated by this system.

It must be noted that the process of calculation of the illuminance of diffusive surfaces and of the surfaces specified by a BDRF is relatively slow. The construction of a realistic image with the help of the backward Monte Carlo ray tracing method takes minutes or sometimes several tens of minutes. For example, generation of fig. 7 with resolution 640x480 pixels required about 3 minutes for Pentium IV 2.66 GHz with 1Gb RAM. At the same time, these calculations and the selection of bright regions are independent of the spatial resolution of the HDRI. Therefore, large HDRIs can be used without additional computational effort.

Fig. 7a. The image generated with original HDRI.

Fig. 7b. The image generated after sun reconstruction.

6 Conclusion

One of the main requirements to the project during its elaboration was physical correctness of simulated illumination. This is why correct representation of the sun in HDRI panoramas was so vital for us. The elaborated algorithms are physically justified. The possibility to calculate the luminance of a point on a surface defined by its BRDF makes it possible to visualize materials with complex optical properties under daylight conditions defined by the shot HDRIs.

The analysis of several tens of HDRIs containing the sky showed that physically correct HDRIs are quite rare. We did not meet them at all but possibly such HDRIs exist. All the analyzed HDRIs have some drawbacks at least in the representation of the sun. Technically the shooting of the sun is a complex problem requiring significant time. Seems in most cases the HDRI producers minimized time of HDRI shooting defying correct acquisition of the sun region. Moreover some of HDRIs do not have a real range of intensities and are just ordinary photographs converted in the HDRI format. Obviously, such HDRIs cannot be used for generation of realistic images.

The elaborated modifications of lighting algorithms take into account the known drawbacks of HDRIs. Both, HDRI drawback compensation and sun reconstruction, improve the quality of the generated images. As a result, the visualization system produces better images and is stable with respect to various errors in HDRIs. This enables us to increase the number of HDRIs that can be used in the synthesis of realistic images.

Acknowledgments

This work was supported by the Russian Leading Scientific Schools Foundation, project no. NSh-2139.2008.9, RFBR, grant no. 07-01-00450, and by INTEGRA Inc. (Japan).

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