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A Model of Pulsed Oscillator Network and the Perspectives of Its Application to the Problems of Information Routing in Wireless Sensor Networks

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Abstract—Development of performance principles of wireless sensor networks (WSN) and the methods of information routing in the networks, providing a capability of self-organized and automatic style of the WSN network work, is currently of substantial interest. The attraction of associated model of pulsed oscillator networks seems to be helpful for the design of adaptive synchronization-based information routing in the WSN. An oscillatory network model with pulsed oscillator dynamics and pulsed oscillator interaction is proposed. The initial version of information transfer in the WSN with the help of synchronization of an associated oscillatory network is discussed.

Keywords: pulsed oscillator networks, synchronization, wireless sensor networks, synchronization-based information routing in WSN

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1. INTRODUCTION

The advances in the field of microelectronics opened the way for creating the new technology – wireless sensor networks (WSN). The WSN are massive heterogeneous spatially distributed densely interconnected networks, containing sensor nodes, sinks and gateway nodes that are mutually coupled via radio communication. Minor consumptive processors, transceivers and miniature sensor chips (MEMS, RFID) are used in WSN as active processing units. The WSN nodes are capable to combine the functions of remote detectors, microcomputers and communication devices. The number of WSN nodes can achieve 10^7. The WSN networks are used in various monitoring problems. They are capable to provide information gathering, initial stage of data processing and information transmission over large spatial regions during long time periods.

As a rule, the WSN networks are self-forming systems, capable to adapt to external environment and to provide intellectual types of information possessing. Their adaptation is usually realized via self-organized modification of the network interconnectivity architecture. The internal network connectivity rules comprised in the WSN networks are capable to provide a multi-hop type of information transmission through the WSN. A dynamically adaptable network interconnectivity architecture also allows to solve such problems as joining additional new nodes to the network, expansion of the spatial area, occupied by the WSN, self-restoring problems (capability to proceed the previous work in case of death of some network nodes) and so on [1, 2].

The WSN networks significantly differ from traditional artificial neural networks, and so the development of special distributed adaptive algorithms of information processing is necessary for the networks. A variety of algorithms of information transmission through WSN (routing protocols, RP) has already been created, reflecting a great diversity of data exchange in WSN. The choice of concrete routing protocol is naturally defined both by the tasks to perform, and by the network interconnectivity architecture type. There exist routing protocols, providing energy efficient WSN work, position-based routing protocols (using the features of spatial WSN location), routing protocols based on arrangement of preliminary network clusterization, routing protocols based on the design of directed information diffusion through WSN, and many others (see, for example, [3–6]). Synchronization-based routing protocols also play a significant role.

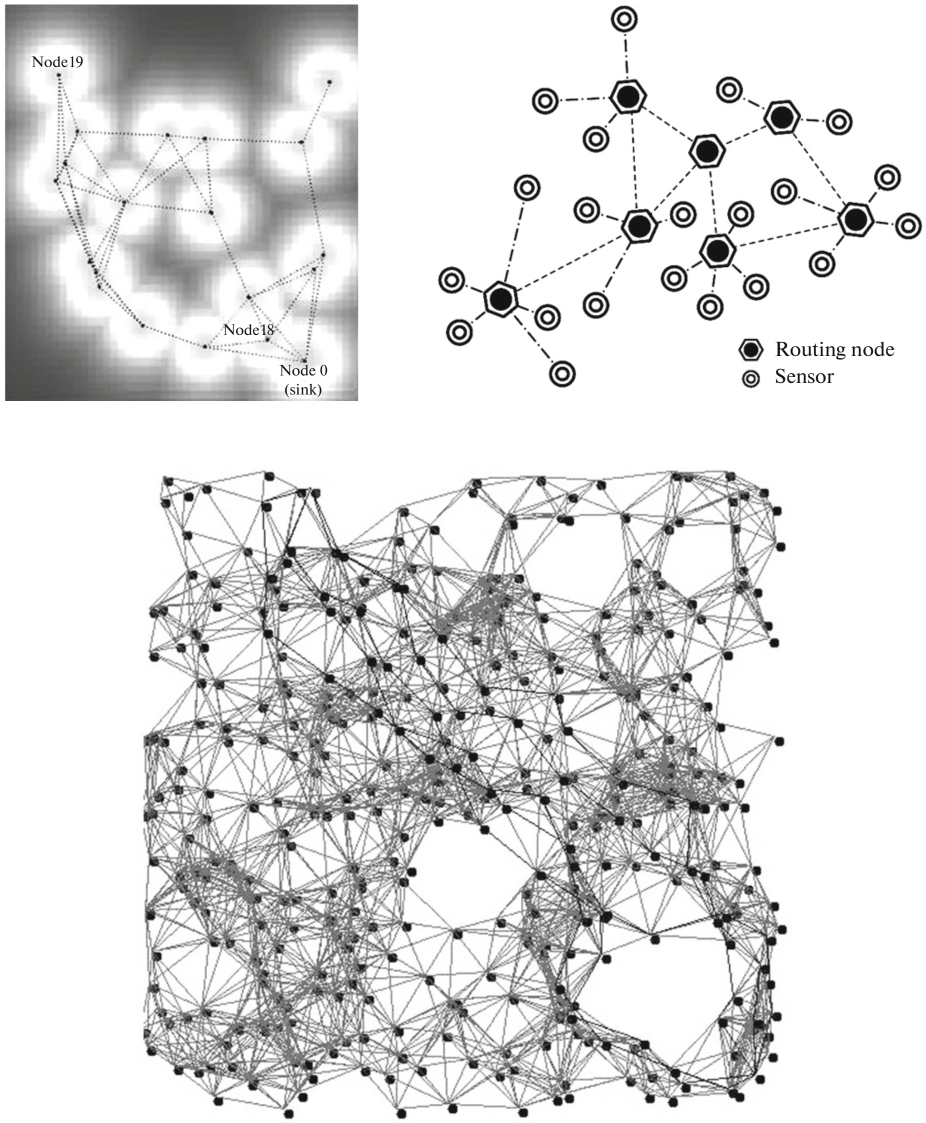

Fig. 1. The examples of WSN networks.

Time synchronization can be used as one of the possible tools of the WSN network self-organized performance [7]. The attention to synchronization processes is caused by the need of frequent change of the network interconnectivity structure. Various synchronization-based routing protocols have been already developed:

- the clusterization-based synchronization protocol (2010);
- the decentralized routing protocol with the guaranteed synchronization level (2010);
- the synchronization protocol for multi-channel data transfer (2011);
- the protocol for WSN node synchronization at the conditions of gradual WSN degradation.

A synchronizable network of phase oscillators with pulsed oscillator interaction has been studied in [8] from the viewpoint of WSN tasks. The synchronization conditions for multi-oscillatory system have been elucidated, the confirming computer experiments being performed.

A model of oscillatory network with pulsed oscillator dynamics and pulsed oscillator interaction is proposed in the present paper. The network oscillator has been designed based on the limit cycle oscillator model developed previously by the authors for problems of image processing. An initial version of network coupling rule, appropriate for designing synchronization-based self-organized data transmissions through

WSN, is suggested. The discussed type of information routing could be viewed as an adaptive mesh-based information transmission based on preliminary clusterization of WSN network.

2. A MODEL OF PULSED OSCILLATOR

The model of limit cycle oscillator, developed previously for image processing problems [9–11], can be written in terms of complex-valued variable $u = u_1 + iu_2$ as:

$$\dot{u} = f(u, I), \tag{1}$$

where

$$f(u) = \left[\rho^2 + i\omega - |u - \rho(1+i)|^2 - \alpha T(\rho)\right](u - \rho(1+i)), \tag{2}$$

$$\rho = \rho(I);\quad T(\rho) = \frac{1}{2}\left[\left|\tanh\left(\sigma(\rho - h_*)\right)\right| - \tanh\left(\sigma(\rho - h_*)\right)\right]. \tag{3}$$

The dynamical system (1–3) possesses the stable limit cycle in the form of the circle of radius ρ, the circle center being located at the point $u_{10} = u_{20} = \rho(1+i)$ of the phase plane (u_1, u_2). For adaptation of the model to WSN problems we introduce the limit cycle radius (the amplitude of auto-oscillations) on some external parameter I, where I is a continuous-valued magnitude, related to some external characteristics, detected by the WSN. For example, it could be a concentration of some poisonous substance. It is supposed, that the dependence of limit cycle radius ρ on I can be expressed by some monotone function $\rho = \rho(I)$. The other parameters, figuring in (1–3), are the following internal parameters of single oscillator dynamics: ω is the frequency of free oscillations of the network oscillator; h_* is the value of the threshold for I, such that at $I < h_*$ the stable limit cycle bifurcates into stable focus; α is a parameter, defining the speed of oscillation damping in the stable focus; σ is a constant, $\sigma \gg 1$.

The pulsed oscillator model is constructed via time-modulation of limit cycle radius ρ of the dynamic system (1–3). The following periodic function $P(t; \Delta, T_0)$ has been used as the modulating function:

$$P(t; \Delta, T_0) = \frac{1}{2}\sum_{k=0}^{K}\left[\tanh\left(\sigma(t - kT_0)\right) - \tanh\left(\sigma(t - kT_0 - \Delta)\right)\right], \tag{4}$$

where T_0 is the parameter, defining the pulse periodicity, and Δ is the parameter, defining the pulse time duration. The dynamical system, governing the dynamics of free pulsed oscillator, can be written in variables (x, y, z), $u = x + iy$, z being the oscillation amplitude. The system can be written in the form:

$$\begin{gathered}\dot{x} = \left[z^2 - (x-z)^2 - (y-z)^2\right](x - z) - \omega(y - z) - \alpha T(z)(x - z),\\ \dot{y} = \omega(x - z) + \left[z^2 - (x-z)^2 - (y-z)^2\right](y - z) - \alpha T(z)(y - z),\\ \dot{z} = \rho_0 \dot{P}(t),\end{gathered} \tag{5}$$

where

$$T(z) = \frac{1}{2}\left[\left|\tanh\left(\sigma(z - h_*)\right)\right| - \tanh\left(\sigma(z - h_*)\right)\right],$$

$$P(t;\Delta) = P_0(t;\Delta) + P_0(t - T_0;\Delta),\quad P_0(t;\Delta) = \frac{1}{2}\left[\tanh(\sigma t) - \tanh(\sigma(t - \Delta))\right], \tag{6}$$

$$\dot{P}(t;\Delta) = \frac{\sigma}{2}\left[\tanh^2(\sigma(t - \Delta)) - \tanh^2(\sigma t) + \tanh^2(\sigma(t - \Delta - T_0)) - \tanh^2(\sigma(t - T_0))\right].$$

(For simplicity we considered here the case of a pair of pulses only).

The oscillator with the dynamics, defined by the system (5)–(6), generates a sequence of two pulses of almost sinusoidal oscillations of the frequency ω through the time interval T_0, the pulse duration being equal to Δ (see Fig. 2). When the sum in the formula (4), defining the function $P(t)$, contains K terms, K identical pulses will be generated with the time period T_0.

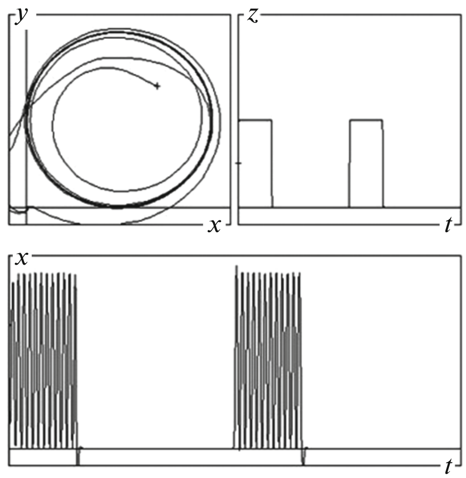

Fig. 2. The dynamics of free pulsed oscillator.

3. TWO COUPLED PULSED OSCILLATORS

If one constructs oscillator interaction that is dependent on oscillator states of each network oscillator pair in a bilinear manner (as it was previously done for oscillatory networks designed for image processing tasks), the oscillator interaction will be pulsed as well. To study the capabilities of the network of the pulsed oscillators with the pulsed interaction, it is useful to consider first the system of two coupled pulsed oscillators. The full network of oscillators, that we intend to use further, is located at the nodes of some two-dimensional spatial grid. The system of ODE, governing the oscillatory network dynamics, can be written in the form

$$\dot{\mathbf{u}}_j = f(\mathbf{u}_j) + \kappa \sum_{j'} W_{jj'} \cdot (\mathbf{u}_{j'} - \mathbf{u}_j), \tag{7}$$

where $\mathbf{u}$ is a three-component oscillator state vector (see eq. (5)). Let the vectors $\mathbf{u}_{1,2}$, defining the states of an oscillator pair, be specified by $\mathbf{u}_1 = (x_1, y_1, z_1)$, $\mathbf{u}_2 = (x_2, y_2, z_2)$, and each of the oscillators generates two successive pulses, which we denote as $P^{(1,2)}(t) = P_0^{(1,2)}(t;\Delta_{1,2}) + P_0^{(1,2)}(t - T_{1,2};\Delta_{1,2})$, where $P_0(t;\Delta)$ is defined in (6). Then the dynamical system (7) for the case of interacting oscillator pair can be written as

$$\begin{aligned}
\dot{x}_1 &= \left[z_1^2 - (x_1 - z_1)^2 - (y_1 - z_1)^2\right](x_1 - z_1) - \omega_1(y_1 - z_1) - \alpha T(z_1)(x_1 - z_1) + \kappa W \cdot (x_2 - x_1), \\
\dot{y}_1 &= \omega_1(x_1 - z_1) + \left[z_1^2 - (x_1 - z_1)^2 - (y_1 - z_1)^2\right](y_1 - z_1) - \alpha T(z_1)(y_1 - z_1) + \kappa W \cdot (y_2 - y_1), \\
\dot{z}_1 &= \rho_1\left[\dot{P}_0(t;\Delta_1) + \dot{P}_0(t - T_1;\Delta_1)\right], \\
\dot{x}_2 &= \left[z_2^2 - (x_2 - z_2)^2 - (y_2 - z_2)^2\right](x_2 - z_2) - \omega_2(y_2 - z_2) - \alpha T(z_2)(x_2 - z_2) - \kappa W \cdot (x_2 - x_1), \\
\dot{y}_2 &= \omega_2(x_2 - z_2) + \left[z_2^2 - (x_2 - z_2)^2 - (y_2 - z_2)^2\right](y_2 - z_2) - \alpha T(z_2)(y_2 - z_2) - \kappa W \cdot (y_2 - y_1), \\
\dot{z}_2 &= \rho_2\left[\dot{P}_0(t;\Delta_2) + \dot{P}_0(t - T_2;\Delta_2)\right].
\end{aligned} \tag{8}$$

For the system (8) the parameters $\omega_{1,2}$ specify the frequencies of free oscillations in the pulses, the parameters $\rho_{1,2}$ define the maximal values of oscillation amplitudes, $\Delta_{1,2}$ define the pulse durations, $T_{1,2}$ is the pulse periodicities, κ (coupling strength) and W are free constant parameters. The functions $P_0(t;\Delta)$ and $\dot{P}_0(t;\Delta)$ in (8) are calculated accordingly to formulas (6).

For an oscillatory network, the parameter W becomes a function, capable to provide cluster network synchronization. It could be chosen dependent on amplitudes $z_{1,2}$ in the form

$$W = W(z_1, z_2) = g(z_1 z_2 - h_1), \quad g(x) = \frac{1}{1 + e^{-\sigma x}}. \tag{9}$$

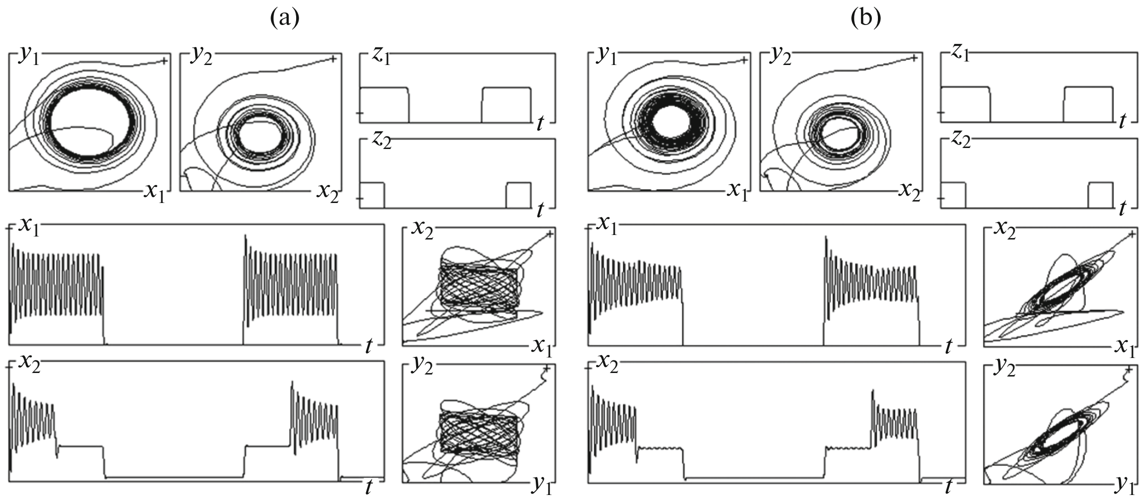

Fig. 3. The dynamics of two coupled pulsed oscillators: (a) the absence of synchronization at $\kappa = 0$; (b) the onset of synchronization at $\kappa = 1$.

The characteristics of dynamics of two coupled pulsed oscillators with $\omega_1 \neq \omega_2$, $\rho_1 \neq \rho_2$, $\Delta_1 \neq \Delta_2$ and $T_1 \neq T_2$ are presented in Fig. 3. Here the behavior of the projections of the dynamical system trajectory onto the planes (x_1, y_1), (x_2, y_2), (x_1, x_2), (y_1, y_2) is depicted. The time-behaviors of variables x_1 and x_2 are shown as well. If the coupling strength κ is sufficiently large, and the pulse durations $\Delta_{1,2}$ of both oscillators are overlapped (so that the number of oscillations during the overlapped time is sufficiently great), the synchronization usually occurs. The synchronization onset can be indicated via the behavior of trajectory projections on the (x_1, x_2) and (y_1, y_2) planes. If the oscillator pair possesses slightly different parameters of internal dynamics (slightly different $\omega_{1,2}$, $\Delta_{1,2}$ and $T_{1,2}$), it is nevertheless possible to achieve the synchronization of the pair by increasing of the interaction strength κ.

4. ON THE POSSIBILITIES OF UTILIZATION OF PULSED OSCILLATOR NETWORKS IN WSN PROBLEMS

In the monitoring tasks where the spatially distributed WSN networks are exploited, huge volumes of sensor information to be processed usually arise. In these situations adaptive, self-organized and automatic style of network performance is necessary. In addition, it is necessary to provide the reliability and energy effectiveness of network functioning. By the reason the simplest types of time synchronization were proposed long ago for providing of WSN work effectiveness. The further development of flexible types of self-organized controllable synchronization for realization of self-organized automatic performance of massive distributed WSN performance seems to be suitable.

The oscillatory network approach based on controllable clusterized synchronization in two-dimensional spatially distributed oscillatory networks has been previously developed for problems of image processing. The approach demonstrated good performance due to the following features of the oscillatory network model: (a) the amplitude of oscillations of single network oscillator was dependent on the brightness of image pixel which the network oscillator corresponds to; (b) the constructed network coupling rule was dependent (via threshold-wise manner) on the oscillation amplitude product of the interacting oscillator pair. These model features provided self-organized oscillatory network decomposition into a collection of internally synchronized, but mutually desynchronized clusters, corresponding to the collection of image brightness fragments [10, 11]. Additionally, the algorithm provided the extraction of all the components of topologically non-connected image brightness fragment corresponding to the same brightness value [11]. The latter algorithm could be useful for those monitoring tasks, implemented by WSN networks, where massive volumes of information have to be processed and where a preliminary clusterization of WSN network would be helpful. One of the ways could be based on the design of oscillatory network model, associated with spatially distributed WSN, and on the development of an algorithm of self-organized oscillatory network synchronization. First of all, the construction of the 2D plane region being in

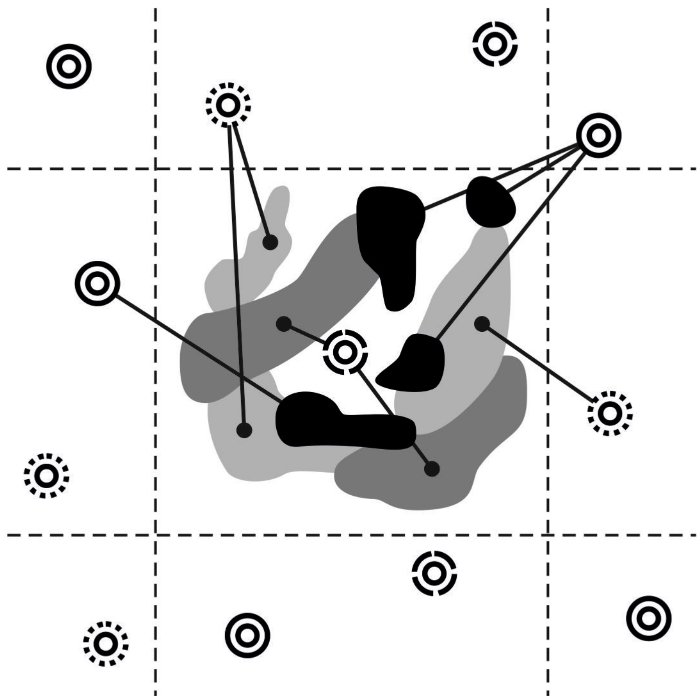

Fig. 4. A simple preliminary version of sensor information transfer through the sinks of WSN network fragment via cluster synchronization in associated oscillator network.

one-to-one correspondence with the area to be monitored by the WSN is required. (For example, the 2D plane region could be constructed as the projection of the monitored area onto a proper 2D plane.) The second step would consist in the specification of a spatial grid in the 2D spatial region and in obtaining of the WSN sensor and sink node projections in the 2D plane domain. The oscillators of the 2D oscillatory network should be located at the cell centers of the 2D plane domain. At last, the proper external characteristics I, related to the data monitored by the WSN network, should be specified (for the design of a parametrical dependence of single oscillator dynamics on I). The initial oscillatory network coupling rule could include the dependence of oscillator interaction on the product of oscillator activities of any interacting oscillator pair.

5. SOME PRELIMINARY SPECULATIONS ON THE DESIGN OF SYNCHRONIZATION-BASED DATA TRANSMISSION THROUGH WSN

So, for construction of oscillatory network, associated with WSN, it is necessary:

(1) To specify the 2D domain for localization of associated oscillatory network (for instance, via using a proper projection of the area to be monitored by the WSN); to specify a 2D grid in the 2D domain; where oscillators of the oscillatory network, associated with the WSN, should be located.

(2) To identify the coordinates of locations of both the WSN sensor nodes and the WSN sinks in the 2D domain.

(3) To specify the parameter I, associated with the data, monitored by the WSN sensors; to choose the scale $I^{(1)},\ldots,I^{(K)}$ for the parameter I.

(4) To design the proper I-parameterization of single oscillator dynamics; to provide synchronization of the oscillatory network for each $I^{(k)}$, $k = 1,\ldots,K$; to find the coordinates of the centers of all synchronized fragments of the oscillatory network, corresponding to each $I^{(k)}$.

(5) To transmit the information received by each synchronized oscillatory network fragment into the closest WSN sink.

(6) To mark all the sinks, containing non-zero information for all $I^{(k)}$. (Only these marked sinks should belong to the ensemble of paths realizing the information transmission by the WSN network).

The sketch of this simplest version of sensor information transmission through the sinks of WSN fragment is schematically depicted in Fig. 4.

6. CONCLUDING REMARKS

The following results are presented in the paper:

• a model of pulsed oscillator has been designed based on the previously developed model of limit cycle oscillator via limit cycle radius time modulation (the dynamical system (5–6));

• an initial version of network connectivity rule for pulsed oscillator networks with pulsed oscillator coupling has been suggested (the expression (8));

• computer experiments, providing the elucidation of the synchronization onset for a pair of coupled pulsed oscillators, have been performed.

The advantage of the designed pulsed oscillator dynamics consists in the possibility of easy parametrical control of the main characteristics of internal dynamics (such as oscillation frequency in the pulse, oscillation amplitude, the pulse duration, the pulse periodicity). As it was verified, the synchronization of interacting oscillator pair is achievable under admissible deviations of internal parameters of dynamics for both oscillators.

The results can be viewed as an initial step in development of synchronization based information routing protocols for 2D spatially distributed WSN networks.

The associated oscillator network model might be also useful for studying the WSN networks as massive complex heterogeneous densely interconnected networks with strongly dynamically variable interconnection architecture. It is worth to be noted that the onset of phase transition (of technogeneous catastrophe type) can be a highly probable event for such kind of complex networks under some "dangerous" switching off a group of network connections.

REFERENCES

1. Berger, R., Introduction to Wireless Sensor Networks, *NI Technical Symposium*, 2009.
2. Hill, J.L., System architecture for Wireless Sensor Networks, *PhD Thesis*, UC Berkeley, 2003.
3. Hadjila, M., Guyennet, H., and Feham, M., A chain-based routing protocol to maximize the lifetime of Wireless Sensor Networks, *Wireless Sensor Network*, 2013, vol. 5, pp. 116–120.
4. Guioufi, A., Nasri, N., Farah, A.B., and Kachouri, A., Clustering algorithm for small-scale Wireless Sensor Networks, *Wireless Sensor Network*, 2013, vol. 5, pp. 67–75.
5. Hamzi, A., Koudil, M., Jamont, J.P., and Occello, M., Multi-agent architecture for the design of WSN application, *Wireless Sensor Network*, 2013, vol. 5, pp. 14–25.
6. Lee, H., Jang, M., and Chang, J.W., A new energy-efficient cluster-based routing protocol using a representative part in Wireless Sensor Networks, *Int. J. Distributed Sensor Networks*, 2014, Article ID 527928.
7. Serpedin, E. and Chaudhan, Q.M., *Synchronization in Wireless Sensor Networks*, Cambridge Univ. Press, 2009.
8. Au, Z. et al., Linear pulsed-coupled oscillators model – A new approach for time synchronization in wireless sensor networks, *Wireless Sensor Network*, 2010, vol. 2, pp. 108–114.
9. Grichuk, E.S., Kuzmina, M.G., and Manykin, E.A., Selective segmentation of real images via oscillatory network approach, *Proc. International Conference "Neuroinformatics 2007"*, 2007, vol. 3, pp. 235–244 [in Russian].
10. Grichuk, E.S., Kuzmina, M.G., and Manykin, E.A., Object selection in visual scene via oscillatory network with controllable coupling and self-organized performance, *Opt. Mem. Neural Networks (Inform. Optics)*, 2011, vol. 20, no. 2, pp. 113–119.
11. Kuzmina, M., Manykin, E., and Grichuk, E.S., *Oscillatory Neural Networks in Problems of Parallel Information Processing*, Berlin/Boston: Walter DeGruyter, 2013.