Spatially Distributed Oscillatory Networks Related to Modelling of the Brain Visual Cortex

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Abstract

Oscillatory networks defined in 3D lattice are designed for modelling of synchronization-based performance of the brain visual cortex. Spatio-temporal dynamics of 1D and 2D oscillatory media, representing continual analogy of the designed networks at some extreme situations, is studied. Confirming computer experiments are in progress.

Introduction

Oscillatory network model initiated in [1] belongs to the class of neuromorphic models exploiting the onset of synchronization of oscillations in oscillatory system.
Synchronous oscillations play essential role in the performance of various brain structures: visual and auditory cortices, olfactory bulb, hippocampus. The ideas concerning possible role of synchronization in visual processing were discussed since 80-th.(C.von der Malsburg, 1985; W.Singer, 1988). After the experimental discovery of synchronous oscillations in the brain visual cortex (VC) in 1988-1989 the evidence exists that synchronization arises in VC due to long-range dynamical connections that are strongly dependent on orientations of the receptive fields (RF).
Series of models was designed to propose biologically motivated synchronization-based versions of VC functioning. There were used both networks consisting of oscillators [2,4] and that ones of spiking neurons [3]. Usually 2D single- and multi-layered architectures with local exitatory and global inhibitory connections were designed.
The model proposed aims at elucidation of the role of synchronization in the visual cortex functioning. We mainly concentrated on the problem of visual image contour detection.
Developing the model, proposed by Z.Li [5], we have designed 3D oscillatory network of columnar architecture that simulates the columnar structure of the VC. It is assumed that single oscillator in VC is formed by a pair of neurons interconnected via excitatory and inhibitory connections. Biologically plausible versions of the oscillator was previously proposed by W.Freeman (1987) and Z.Li & J. Hopfield (1989) in odor segmentation problem.
In our model more symmetrical limit-cycle oscillator in the vicinity of Hopf bifurcation is used as network oscillator. The bifurcation parameter is appropriately controlled by visual stimulus properties — local contrast and bar orientation. As a result the oscillator demonstrates sharp stimulus-dependent intrinsic dynamics — stable oscillations (limit cycle of sufficient size) or "silence" (stable focus, providing quickly damping oscillations). Self-organized dynamical interaction of oscillators has been constructed in factorized form, including the dependence of threshold character on product of instantaneous oscillator activities, orientations of receptive fields and spatial distance between the oscillators in the network. The network dynamics imitates collective behavior of simple cells of VC in visual image contour detection task.
For flexible performance of the model in a wide variety of situations proper tuning of all network parameters is necessary. For the purpose, qualitative mathematical analysis of the main properties of the model is started. The reduced oscillatory network defined in 2D spatial lattice and obtained from 3D one by special kind of inter-column ave-

raging [1], deliver the appropriate object for mathematical study. The averaged network is further reduced to 2D homogeneously locally connected oscillatory network consisting of standard limit-cycle oscillators in the vicinity of Hopf bifurcation. Spatio-temporal dynamics of these networks is closely related to the dynamics of spatially continual nonlinear media governed by nonlinear diffusion equations (reaction-diffusion equations).

The present study concerns the analysis of spatio-temporal regimes in 1D and 2D oscillatory media corresponding to the reduced locally connected oscillatory networks.

Oscillatory Network Model of the VC. The 2D Reduced (Averaged) Network

The oscillatory network model of the VC is designed as the network of columnar architecture consisting of N^2 columns of K oscillators each ($N^2 \cdot K$ is the total number of oscillators). The bases of the columns are located at the nodes of 2D square lattice G_{N^2}, whereas oscillators of each column are located at the nodes of 1D lattice L^K oriented normally with respect to the plane of G_{N^2}. So the oscillators of the whole network are located at the nodes of 3D lattice $G_{N^2} \times L^K$. The location of a single oscillator is specified by a radius-vector $\mathbf{r}_{lm}^k = (x_{jm}^k, y_{jm}^k, z_{jm}^k)$. The state of the network is specified by $(N \times N \times K)$-matrix of oscillator states $[u_{jm}^k]$. For each oscillator the orientation of its RF is specified by 2D unit vector $\mathbf{n}_{jm}^k$, which is an important internal parameter of network oscillator. The retina is modelled by 2D square lattice similar to G_{N^2}. So, a continuous visual image arising in real retina is represented by its discretization in the retina lattice, that is, by a collection of pairs $(I_{jm}, \mathbf{s}_{jm}), j = 1, \ldots, N, m = 1, \ldots, N$, where I_j is local image contrast and $\mathbf{s}_{jm}$ — local orientation of image elementary bar.

We do not discuss so far the way of extracting the data $(I_{jm}, \mathbf{s}_{jm})$ from visual images of continuous spatial structure. It should be related to the methods of hierarchical spatial filtering of multi-scale visual patterns. In the initial stage we instead restrict our consideration by simple images of single-scale spatial structure. Further some versions of pattern processing methods providing spatial filtering are planned to be included.

The internal dynamics of a single oscillator is designed in a manner to imitate stimulus orientation-dependent response of simple cell in the VC. Suitable type of dynamics can be delivered by oscillator with two degrees of freedom which state is defined by two-component real-valued vector function $\mathbf{u} = (u_1, u_2)^\top$. The designed system of two coupled ODE for u_1, u_2 can be written in the form of single equation for complex-valued function $u = u_1 + iu_2$:

$$\dot{u} = (\rho^2 + i\omega - |u-c|^2)(u-c) + \mu_0(g(I-h) + q(\mathbf{s}, \mathbf{n})) \tag{1}$$

Here ρ, c, ω are constants defining asymptotic parameters of limit cycle of dynamical equation (1): at $\mu_0 = 0$ the limit cycle is the circle of radius ρ with center location at the point $c = |c|e^{i\alpha}$ in the complex u-plane, ω is the cycle frequency. The constant μ_0 is some complex (tuning) constant. Suitably constructed functions g and q provide controlling of bifurcation parameter $\mu = (g + q)$: the Hopf bifurcation occurs at some $\mu = \mu_*$, $\mu_* \in (0, 1)$ (at $\mu = \mu_*$ limit cycle is converted into stable focus located in the vicinity of the origin). The dynamical system governing the dynamics of the oscillatory network can be written in the form:

$$\dot{u}_{jm}^k = f(u_{jm}^k, \mu) + S_{jm}^k; \quad j = 1, \ldots, N,$$
$$m = 1, \ldots, N, \quad k = 1, \ldots, K. \tag{2}$$

Here $f(u, \mu) = (\rho^2 + i\omega - |u - c|^2)(u - c) + \mu_0\mu$, $\mu = (g(I - h) + q(\mathbf{s}, \mathbf{n}))$ and the term S_{jm}^k specifies interaction between oscillators in the network. It can be written as

$$S_{jm}^k = \sum_{j', m', k'} W_{jj'mm'}^{kk'}(u_{jm}^k, u_{j'm'}^{k'})(u_{j'm'}^{k'} - u_{jm}^k), \tag{3}$$

where the elements of matrix of connections W are represented in the factorized form:

$$W_{jj'mm'}^{kk'}(u, u') = P_{jj'mm'}^{kk'}(u, u')Q_{jj'mm'}^{kk'}(\mathbf{n}, \mathbf{n}')\times$$
$$D_{jj'mm'}^{kk'}(|\mathbf{r} - \mathbf{r}'|), \tag{4}$$

$\mathbf{r}$ and $\mathbf{r}'$ are radius-vectors, defining spatial locations of oscillators (j, m, k) and (j', m', k').

The 2D reduced network corresponding to the columnar one is defined in lattice G_{N^2} and consists of idealized oscillator-columns. It can be derived as a result of inter-columnar averaging of columnar network and special limit analogous to well-known thermodynamical limit in statistical physics. Its state is defined by $N \times N$ matrix $[u_{jm}]$. The RF orientation $\mathbf{n}_{jm}$ of its single oscillator coincides with the stimulus bar orientation $\mathbf{s}_{jm}$. The internal dynamics of the oscillator is governed by eq.(1) with $q(\mathbf{s},\mathbf{n}) \equiv \mathbf{1}$. Further reduction can be achieved in the case of the averaged network response on special visual stimulus — homogeneous field of sufficient contrast with monodirected bar orientations. Then we have $g(I-h) \equiv 1$. In addition, if we consider the network with pure local interaction, we should put in eq.(4) $P = Q \equiv 1, D^{kk'}_{jj'mm'} = d \cdot \delta_{jj^*}\delta_{mm^*}\delta^{kk^*}$, where $d = const$ and $(j^*, m^*, k^*) \in \mathcal{N}^k_{jm}$, $\mathcal{N}^k_{jm}$ denotes the set of closest neighbors of the oscillator located at the site (j, m, k). As a result we obtain the system of dynamical equations for locally connected oscillatory network of idealized columns defined in G_{N^2}.

Finally, we can relate the obtained system to the following transformed system:

$$\dot{u}_{jm} = (\lambda + i\omega_{jm} - |u_{jm}|^2)u_{jm} + \\ + d \sum_{j'm' \in \mathcal{N}_{jm}} (u_{j'm'} - u_{jm}), \quad j = 1, \ldots, N, \\ m = 1, \ldots, N, \tag{5}$$

where λ is bifurcation parameter and $d = \kappa e^{i\chi} = d_1 + id_2$ is local coupling strength in the network. Here the normal form of oscillator dynamics in the vicinity of Hopf bifurcation is used.

Spatio-Temporal Dynamical Regimes in Oscillatory Media Related to Locally Connected Oscillatory Networks

While studying the dynamics of locally connected networks it is quite helpful to attach their continual analogies — nonlinear media governed by reaction-diffusion equation (RDE) because there is very rich experience of their qualitative mathematical analysis. In our case we should replace the matrix $[u_{jm}]$ by a complex-valued function $u(x,t) = u_1 + iu_2$ depending on spatial variable x, $x \in [0,l] \times [0,l] \subset R^2$. Then RDE governing the dynamics of oscillatory media corresponding to oscillatory networks with dynamics (5) can be written in terms of real-valued two-component vector-function $\mathbf{u} = (u_1, u_2)^\top$:

$$\mathbf{u}_t = \hat{F}(\mathbf{u})\mathbf{u} + \hat{D}\triangle\mathbf{u}, \tag{6}$$

where

$$\hat{F}(\mathbf{u}) = \begin{bmatrix} \lambda - u_1^2 - u_2^2 & -\omega \\ \omega & \lambda - u_1^2 - u_2^2 \end{bmatrix}$$

$$\hat{D} = \begin{bmatrix} d_1 & -d_2 \\ d_2 & d_1 \end{bmatrix} \tag{7}$$

and $\triangle$ is 2D Laplacian. We also assumed here (at the initial stage of study) that $\omega(x) \equiv \omega = const.$ As well known, numerous investigations are devoted to various models of nonlinear active media. A considerable scope of studies concerning oscillatory media also exists, including strict results of qualitative analysis, results of physical level and those of computer modelling. In various situations the media demonstrate familiar collection of spatio-temporal regimes: wave trains, rotating spiral waves, standing waves, targets and shock structures, stripe patterns, cluster states. From the viewpoint of proposed oscillatory network model the media defined in compact spatial domains (finite interval in 1D case, square in 2D case) are of the main interest. The elucidation of conditions leading to arising of spatio-inhomogeneous regimes in the oscillatory media is quite helpful for consistent choice of all the parameter ranges providing correct performance of the oscillatory network.

Mention the features of oscillatory networks to be analyzed: a) local oscillatory dynamics is considered in the vicinity of Hopf bifurcation; b) the interaction is complex-valued (nondiagonal diffusion operator).

The natural initial step of analysis of the types of spatio-temporal dynamics of RDE (6) consists in consideration of bifurcations of spatially homogeneous solution. As well known [7,10], the condition of destabilization of trivial steady-state solution is defined by the spectrum of the operator $\hat{G}(\lambda, \sigma_k) = \hat{L} - \sigma_k^2\hat{D}$, where L is the linearization of $\hat{F}$ around steady-state (the Jacobi matrix) and $\{-\sigma_k^2\}$ is the spectrum of 1D scalar diffusion operator ($\sigma_k = \pi k/l$, $k = 0, 1, 2 \ldots$). The eigenvalues ν_k of $\hat{G}(\lambda, \sigma_k)$ for RDE (6) in 1D case can be easily calculated in explicit form:

$$\nu_k(G) = \lambda - \sigma_k^2 d_1 \pm i(\omega - \sigma_k^2 d_2). \tag{8}$$

As it follows from (8), the diffusion provides stabilizing effect at $d_1 > 0$ and leads to destabilization at $d_1 < 0$; the value d_2 influences only the frequency of oscillations ω.

The analysis of diffusion destabilization can be performed by using different approaches developed [6-9]. In particular, the method developed in [10] can be used as one of reliable methods. It consists in the expansion of RDE solutions into the series on orthonormalized system of eigenfunctions $\{X_m(x)\}$ of scalar diffusion operator:

$$u_1(x,t) = \sum_{m=1}^{\infty} X_m(x) P_m(t),$$
$$u_2(x,t) \sum_{m=1}^{\infty} X_m(x) Q_m(t). \quad (9)$$

The functions X_m are defined by boundary conditions for RDE: $X_m(x) = \sin(\sigma_m x)$ at $u_1(0,t) = u_2(l,t) = 0$ and $X_m(x) = \cos(\sigma_m x)$ at $u_{1x}(0,t) = u_{2x}(l,t) = 0$, $\sigma_m = \pi m/l$. The system of coupled ODE for $\{P_m(t), Q_m(t)\}$, similar to that one derived in [10] for the case $\lambda = 1$, can be obtained for RDE (6) at arbitrary λ. Using the expansion (9) one can extract the ODE defining time behavior of k-th spatial harmonics.

The system of ODE for $\{P_m(t), Q_m(t)\}$ provides also the analysis of existence of standing waves in 1D oscillatory media (special RDE solutions with separated variables x and t). In the case of boundary conditions $u_{1x}(0,t) = u_{2x}(l,t) = 0$ standing waves are the solutions of the form

$$\mathbf{u}(x,t) = \mathbf{U}_0 e^{-i\omega t} + \mathbf{U}_k e^{-i(\omega t + \gamma)} \cos(kx) \quad (10)$$

The existence of standing waves can be established by direct substitution of (10) into the RDE. In this way one can obtain four equations: two equations for $|\mathbf{U}_0^2|$, $|\mathbf{U}_k^2|$, the dispersion equation reflecting the relation between ω and k and the algebraic equation for $\tan(\gamma)$. Analysis of the algebraic equation shows the existence of real-valued solutions for $\tan(\gamma)$. Therefore, standing wave solutions to RDE (6) in 1D case exist. The parametric domain of their existence still remains to be revealed.

Cluster states are RDE solutions with separated variables of another type: they correspond to medium decomposition into synchronously oscillating subdomains (clusters). The own amplitude, phase shift and frequency of oscillations are inherent to each cluster. These regimes are of special interest in the context of the network model.

At last it should be noted that in 2D case RDE (6) with scalar diffusion operator ($d_2 \equiv 0$) belongs to the class of $(\lambda - \omega)$-systems studied in [7,8]. Therefore, all the regimes obtained in [7,8] (target patterns, spiral waves, shock structures) are inherent in RDE (6) in 2D case.

Computer experiments on unclosed oscillatory chains and 2D lattice networks that could confirm the results on the oscillatory media are currently in progress.

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