Synchronization-Based Network Model for Contour Integration in Visual Processing

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Abstract

The oscillatory network is proposed for modelling synchronization-based processing in the primary visual cortex. We concentrate on one of the fundamental problems of visual processing — contour integration in visual image perception. The network performance imitates self-organized collective behavior of simple cells in the visual cortex while realizing the ability of contour integration task. A single oscillator is a limit-cycle oscillator parametrically dependent on visual stimulus properties — local contrast and bar orientation. The oscillator demonstrates sharp stimulus-dependent intrinsic dynamics, imitating simple cell responses — stable oscillatory activity or silence (quickly damping oscillations).

The designed columnar architecture of the network reflects the columnar structure of the visual cortex. The dynamical connection of oscillator pair depends on the product of oscillator activities, orientations of receptive fields (internal parameter of each oscillator) and the distance between the oscillators in the network.

The model demonstrates self-organized synchronization of contour-dependent oscillator ensembles of the network and self-controlled suppression of noisy background.

The related reduced oscillatory network is obtained by averaging over the oscillators inside the columns.

1 Introduction

Network models exploiting the onset of synchronized ocillations represent new class of models for elucidation of the role of synchronization in brain information processing.

Synchronous oscillations were experimentally observed in the visual cortex (VC) of cat and monkey [1,2] and later were discovered in other brain areas. The evidence exists that in primary VC synchronization arises due to long-range dynamical connections extended over distances covering several orientation columns. The emerging dynamical connections are strongly dependent on orientations of receptive fields (RF).

Series of attempts were enterprised to elucidate the role of cortical oscillations and synchronization in visual image processing [3-15]. The oscillatory network model of cluster structure [9] should be mentioned first. The simplest phase oscillator was used as a network processing unit. A cluster of oscillators imitated a column of the VC. RF orientations were assumed to be uniformly distributed over a cluster and short-range stationary inter-cluster connections — dependent on RF orientations. The averaged RF orientation for a single cluster was evaluated in mean-field approximation of statistical physics. The weak long-range inter-cluster interaction, providing synchronization, was calculated. The model was studied in the context of visual image segmentation problems. In particular, some results concerning image contour integration were obtained.

Oscillatory network models for image segmentation were designed in the series of papers [12-14]. Various types of limit cycle oscillators, imitating a whole VC column, were used as processing units. Network architecture with local excitatory and global inhibitory connections was designed. The networks were successfully applied to gray-scale real-world image recognition.

Oscillatory network models of spiking neurons (of homogeneous and two-layered architecture) were used in [9-11] to perform pattern segmentation and feature linking problems. In appropriate parametrical domains two-layered networks demonstrated stationary dynamics, synchronized oscillations and so-called weakly locked states. In addition to computer simulations the analytical results in terms of evolution of macrovariable "overlap" were obtained. In particular, the condition of synchronization and the period of synchronized oscillations were found.

The oscillatory network of columnar architecture imitating functioning of the primary VC in contour integration task was designed in [15]. A single oscillator formed by a pair of excitatory and inhibitory neurons was previously proposed by W.Freeman (1987) and Z.Li & J. Hopfield (1989) in odor segmentation problem. Excitatory-excitatory and excitatory-inhibitory connections were separately specified for all oscillators to achieve biological plausibility. It was shown that synchronization of network ensembles is facilitated for smooth, long and closed contours.

Our model was inspired primarily by the paper [15] and focused on study of synchronization of oscillatory network ensembles dynamically encoded by visual image contours of sufficient contrast.

2 Columnar Oscillatory Network Model

Thus we concentrate on the VC ability of image contour integration. Dealing with this problem it is relevant to model VC as a network of coupled oscillators. Following [12-15], we suppose that single network oscillator is formed by a pair of interconnected excitatory and inhibitory neurons. Stationary network connections of oscillators are supposed to be quasi-local; long-range dynamical connections emerge in the process of oscillatory network functioning and depend on RF orientations.

We design the network of columnar architecture consisting of $N = n^2$ columns of K oscillators each ($N \cdot K$ is the total number of oscillators). The bases of the columns are located at the nodes of 2D square lattice G_N, whereas ocillators of each column are located at the nodes of 1D lattice L_K oriented normally with respect to the plane of G_N. So the oscillators of the whole network are located at the nodes of 3D lattice in the form of parallelogram $ABCDA'B'C'D'$ (see Fig.1). If one chooses the Cartesian coordinates with the origin at the point A and the axes x, y, z oriented along AD, AB and AA' respectively, the location of a single oscillator can be obviously specified by vector $\mathbf{r}^k_{lm} = (x^k_{lm}, y^k_{lm}, z^k_{lm}) \equiv (a \cdot l, \quad a \cdot m, \quad b \cdot k), \quad l = 1, \dots n, \quad m = 1, \dots n, \quad k = 1, \dots K, \quad n^2 = N,$ where a and b are lattice periods of G_N and L_K respectively. The state of columnar network then is naturally specified by $(n \times n \times K)$-matrix of oscillator states.

We also will use further the successive numeration of the nodes of G_N, utilizing a pair of indices $(j, k), \quad j = 1, \dots N; \quad k = 1, \dots K$ for specification of network oscillator locations. In the case the state of the whole network is specified by $(N \times K)$-matrix. It permit to simplify formulas defining contribution from oscillator interaction.

For each oscillator in a column the orientation $\mathbf{l}^k_j$ of its RF is specified ($\mathbf{l}^k_j$ is 2D unit vector, $\mathbf{l}^k_j = (\cos \psi^k_j, \sin \psi^k_j)$, $\psi^k_j \in [0, \pi)$). The orientations $\mathbf{l}^k_j$ are assumed uniformly distributed: $\psi^k_j = k\pi/K, k = 1, \dots, K, j = 1, \dots, N$. Homogeneous distribution of RF orientations corresponds to regular arrangement of the corresponding layers inside the VC [16]. The orientation $\mathbf{l}^k_j$ of receptive field is an important internal parameter of the network oscillator.

Fig.1

Fig.1

A visual image formed on retina represents an input into the oscillatory network. Model retina should be considered as 2D square lattice completely structurally equivalent to G_N. Therefore, network input can be naturally parametrized by pairs of parameters $(I_j, \mathbf{s}_j), j = 1, \ldots, N$, where I_j is local contrast and $\mathbf{s}_j$ — local orientation of elementary image bar. It is assumed that the same input $(I_j, \mathbf{s}_j)$ is delivered to all oscillators of the j-th column.

An input from retina is defined an initial state of recurrent oscillatory network. Being initiated by some input the network state transfers into some state of stationary synchronized activity, where only appropriate subensemble of network oscillators undergoes synchronized oscillations.

In the case of adequate network functioning the problem of contour integration should be solved with the help of proper projection of the synchronized subensemble into G_N. We try to achieve this aim by the combination of two network features: stimilus-tuned single oscillator dynamics and special type of nonlinear self-controlled dynamical interaction of network oscillators.

2.1. Single Columnar Oscillator.

The internal dynamics of a single oscillator is tuned in a manner to imitate stimulus orientation-dependent response of simple cell in VC (Fig.1a,b). Namely, an oscillator is in active regime (nondamping oscillations) if the following conditions are simultaneously satisfied:
a) $I(\mathbf{r}) \geq h_0$, where h_0 is some threshold value (imitates noise level contrast);
b) the orientation $\mathbf{l}$ of RF of the oscillator is sufficiently close to the orientation $\mathbf{s}$ of the corresponding visual image bar.
The oscillator transfers into passive, or silent state (quickly damping oscillations or relaxation), if at least one of these conditions is not satisfied.

The system of two coupled differential equations for free columnar oscillator can be written in the form of differential equation for complex variable $z = x + iy$:

$$\dot{z} = (\rho^2 + i\omega - |z - c|^2)(z - c) + \lambda(1 - \Gamma(|\psi - \beta|) + (1 - g(I - h_0)) \quad (1)$$

Here ρ, c, ω are constants defining asymptotic parameters of stimulus-dependent limit cycle of dynamical equation (1): at $\lambda = 0$ the limit cycle is the circle of radius ρ with center location at the point $c = |c|e^{i\alpha}$ in the complex plane, ω is the cycle frequency. The complex-valued constant λ is a tuning constant. The function $\Gamma(|\phi|)$ is a symmetrical peak-shaped function with maximum at $\phi = 0$. We used the function $\Gamma(|\phi|) = 2e^{-\sigma|\phi|}/(1 + e^{-2\sigma|\phi|})$. Parameter σ controls the width of $\Gamma(|\phi|)$. Sigmoidal threshold function g can be also chosen of controlled steepness: $g(x - h_0) = 1/(1 + e^{-2\nu(x-h_0)})$.

Note that under proper scaling the variables $x = Re(z)$ and $y = Im(z)$ can be interpreted as the states of excitatory and inhibitory neurons forming the oscillator.

2.2 Dynamical Equations for the Columnar Oscillator Network.

The state of the columnar network can be specified by $N \times K$-matrix $\hat{z} = [z_j^k]$, where j is the number of orientation columns, $j = 1, \ldots, N$, and k is the number of oscillator in the column, $k = 1, \ldots, K$. Then the dynamical system governing the dynamics of the network can be written in the form:

$$\dot{z}_j^k = (\rho^2 + i\omega - |z_j^k - c|^2)(z_j^k - c) + \lambda(1 - \Gamma(|\psi_j^k - \beta_j|) + (1 - g(I_j - h_0)) + S_j^k; \; j = 1, \ldots, N, k = 1, \ldots, K. \quad (2)$$

The term S_j^k determines interaction between oscillators in the network. If we denote by $\mathcal{N}_j^k$ the set of neighbors of an oscillator numbered (j, k) that are connected with it, the interaction term S_j^k can be written as

$$S_j^k = \sum_{(j',k') \in \mathcal{N}_j^k} W_{jj'}^{kk'}(z_j^k, z_{j'}^{k'}) z_{j'}^{k'}. \quad (3)$$

The connection weights $W_{jj'}^{kk'}(z_j^k, z_{j'}^{k'})$ depend in nonlinear manner on the oscillator states z_j^k and $z_{j'}^{k'}$. In our model the following dependencies of $W_{jj'}^{kk'}$ are included:

- the threshold character of the dependence on the product of mean levels of oscillator activities;
- the dependence on RF orientations;
- the dependence on the distance between the oscillators in the network.

Namely,

$$W_{jj'}^{kk'}(z_j^k, z_{j'}^{k'}) = g(w|z_j^k||z_{j'}^{k'}| - h) \times F(\mathbf{l}_j^k, \mathbf{l}_{j'}^{k'}) D(|\mathbf{r}_j^k - \mathbf{r}_{j'}^{k'}|) e^{i\chi_{jj'}^{kk'}}. \quad (4)$$

Here $\mathbf{r}_j^k$ and $\mathbf{r}_{j'}^{k'}$ are radius-vectors, defining geometrical locations of oscillators (j,k) and (j',k') and $|z| = (x^2+y^2)^{1/2}$. The function $F(\mathbf{l}_j^k, \mathbf{l}_{j'}^{k'}) \equiv F(|\psi_j^k - \psi_{j'}^{k'}|)$, defining the dependence of interaction on RF orientations, is similar to $\Gamma(|\phi|)$ in eq.(1). The angle $\chi_{jj'}^{kk'}$ determines the phase of the connection weight.

3 Reduced Oscillatory Network.

In the columnar oscillatory network all the $N \times N$ matrices of connections $[W_{jj'}^{kk'}]$ must be specified. Given the interaction $[W_{jj'}^{kk'}]$ and all response functions Γ, the inter-column averaging over the oscillators in each column can be carried out. In this way we obtain the network consisting of averaged columns defined in 2D grid G_N. Such reduced network can be naturally regarded as a macro-level approximation of the columnar (micro-level) network. The inter-columnar averaging becomes trivially simple in a special limit that is the analogy of well-known thermodynamical limit in statistical physics. Indeed, consider the following continual limit: a) the column is infinitely long $K \to \infty$; b) the set of angles $\psi^k(\mathbf{l}^k) = (\cos\psi^k, \sin\psi^k)$ tends to continuous distribution over the circle; c) the width of function $\Gamma_K(|\alpha|)$ tends to zero at $K \to \infty$.

The averaged response of the idealized column obviously coincides with the response of single columnar oscillator that possesses RF orientation $\mathbf{l} \equiv \mathbf{s}$. As a result we obtain the reduced oscillatory network consisting of oscillator-columns defined in a plane grid G_N. Its state is defined by N-dimensional vector $z = (z_1, \ldots, z_N)$ instead of $N \times K$ matrix $\hat{z}$. The orientation $\mathbf{l}_j$ of its single j-th oscillator coincides with the stimulus bar orientation $\mathbf{s}$; the internal dynamics of the oscillator is governed by eq.(1) with $\Gamma(|\psi - \beta|) \equiv 1$.

Dynamical equations of the reduced network are:

$$\dot{z}_j = (\rho_j^2 + i\omega_j - |z_j - c|^2)(z_j - c) + \lambda(1 - g(I_j - h_0)) = \sum_{k=1}^{N} W_{jk}(z_j, z_k; \mathbf{s_j}, \mathbf{s_k}; h) z_k, \; j = 1, \ldots, N, \quad (5)$$

where

$$W_{jk} = g(w|z_j||z_k| - h) F(\mathbf{s_j}, \mathbf{s_k}) D(|\mathbf{r_j} - \mathbf{r_k}|) e^{\chi_{jk}}$$

4 Computer Experiments

The initial series of computer experiments was carried out with reduced oscillatory network. The first step was to design properly tuned dynamics of single oscillator. The following properties were achieved:

a) at $I \geq \bar{I}$ the limit cycle of eq.(1) is a circle of maximum radius and maximum distance from zeropoint;

b) under decreasing I the cycle radius is sharply decreasing;

c) the limit cycle bifurcates into stable focus (or node) located at the vicinity of zero at $I \leq I_*$, Fig. 1a,b.

The second step was testing the abilities of interaction (4). It was found out that the interaction provides the desirable self-controlled coupling. Namely, it becomes weak in the cases:

a) one of the oscillator activities $|z_1|, |z_2|$ is close to background;

b) the RF orientations are not close to each other;

c) the distance between oscillator locations exceeds the radius of interaction.

The example of two-oscillator dynamics is shown in Fig.4, where two phase portrait projections and time dependencies $x_1(t), x_2(t)$ are depicted. At the absence of coupling the first oscillator has the limit cycle shown in (x, y)-plane, whereas the second one has the stable node denoted by P in (z, u)-plane. When the coupling is switched on, synchronization occurs if coupling strength w exceeds the threshold value $w_*(w_* = 0.6)$. The limit cycle of small size arises in (z, u)-plane, and relaxational dynamics is changed into oscillations.

The last series of experiments was concerned to 1D reduced network. Synchronization of ensembles encoded by 1D contour of sufficiently high intensity and slowly varying stimulus bar orientations was observed.

The computer experiments allows to expect the promising features of the designed model: background suppression; unambiguous synchronization of oscillator ensembles dynamically encoded by contours of appropriate contrast.

Conclusive Remarks

The columnar oscillatory network model is proposed.

Each oscillator in the network is a limit-cycle oscillator parametrically dependent on visual stimulus contrast and orientation. It demonstrates stimulus-dependent dynamical behavior — oscillatory activity or silence.

Oscillatory connections depend on oscillatory activities, RF orientations, and distances between the oscillators. The designed interaction provides dynamical self-organization of the interconnection architecture of the network.

The reduced oscillatory network of idealized averaged columns has been proposed.

Contour integration has been observed in the initial series of computer experiments.

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