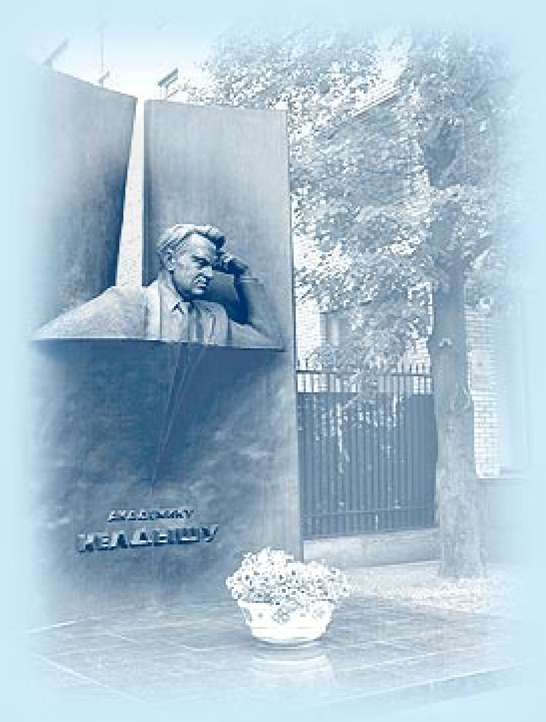

ISSN 2071-2898 (Print)
ISSN 2071-2901 (Online)

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Recommended form of bibliographic references: Kolesnichenko A.V. Modification alpha formalism of Shakura–Sunyaev for the coefficient of turbulent viscosity in an astrophysical disk of finite thickness // Keldysh Institute Preprints. 2022. No. 1. 32 p. **https://doi.org/10.20948/prepr-2022-1-e**
https://library.keldysh.ru/preprint.asp?id=2022-1&lg=e

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Modification alpha formalism of Shakura–Sunyaev for the coefficient of turbulent viscosity in an astrophysical disk of finite thickness

Москва — 2022

Колесниченко А.В.

Модификация альфа-формализма Шакуры–Сюняева для коэффициента турбулентной вязкости в астрофизическом диске конечной толщины.

В приближении одножидкостной гидродинамики сформулирована замкнутая система осредненных по Фавру магнитогидродинамических уравнений, предназначенная для численного моделирования сжимаемых турбулентных течений электропроводных сред в присутствии магнитного поля. Особое внимание уделено методу получения в рамках необратимой термодинамики определяющих соотношений для турбулентного потока тепла и суммарного (кинетического плюс магнитного) тензора турбулентных напряжений. Предложен новый подход к моделированию коэффициента турбулентной кинематической вязкости для астрофизического диска, который учитывает влияние внешнего и генерируемого магнитного поля, а также процессов конвективного переноса тепла на турбулентность в стратифицированном слое конечной толщины и тем самым модифицирует альфа формализм Шакуры–Сюняева, разработанный для тонкого диска и широко используемый в астрофизической литературе.

Ключевые слова: магнитная гидродинамика, развитая турбулентность, термодинамика необратимых процессов, альфа-диски.

Aleksandr Vladimirovich Kolesnichenko

Modification alpha formalism of Shakura–Sunyaev for the coefficient of turbulent viscosity in an astrophysical disk of finite thickness.

In the approximation of one-fluid hydrodynamics, a closed system of Favre-averaged magneto-hydrodynamic equations is formulated, intended for the numerical simulation of compressible turbulent flows of electrically conductive media in the presence of a magnetic field. Special emphasis is paid to the method of obtaining, within the framework of irreversible thermodynamics, the constitutive relations for the turbulent flux heat and the total (kinetic plus magnetic) tensor of turbulent stresses. A new approach to modeling the coefficient of turbulent kinematic viscosity for an astrophysical disk is proposed, which takes into account the influence of an external and generated magnetic field, as well as the processes of convective heat transfer on turbulence in a stratified layer of finite thickness, and thereby modifies the Shakura–Sunyaev alpha formalism developed by for a thin disk and widely used in the astrophysical literature.

Key words: magnetic hydrodynamics, the advanced turbulence, thermodynamics of the irreversible processes, alpha disks.

Introduction

A noticeable fraction of gas in the near-solar protoplanetary disk at the very initial stage of its evolution represents partially ionized plasma; the degree of ionization of this plasma is quite sufficient for the development of various plasma instabilities (see, e.g., [Sano et al. 2000]), in particular, the hydro magnetic shear instability discovered by Velikhov [Velikhov 1959]. This instability, as applied to astrophysical disks, was called Balbus–Hawley magneto rotational instability [Balbus, Hawley 1991]; it occurs if there exists a magnetic field component perpendicular to the disk rotation plane, and the angular velocity of rotation decreases with increasing distance. As a result, a large amount of unstable small-scale (as compared to the disk thickness) modes appear, and the development of these modes effectively generates turbulence in the differentially rotating disk (see, e.g., [Eardley, Lightman 1975; Alfven, Arrhenius 1979; Galeev et al. 1979; Coroniti 1981; Tout, Pringle 1992; Brandenburg et al. 1996; Lesche 1996; Bisnovatyi–Kogan, Levelace 2001; Armitage et al. 2001]).

The existence of a magnetic field (even a weak one, $\left|\mathbf{B}\right|^2 / 4\pi\mu\rho \le c_s^2 4\ \pi\mu\rho$) considerably complicates hydrodynamic flows in the gravitational field of the protostar. The large-scale magnetic forces acting on the conducting layers of the disk noticeably influence the dynamics of the astrophysical processes taking place in the disk, such as the angular momentum transfer to the disk periphery, the character and rate of accretion from the ambient space (from the diffusion medium or some mass-losing satellite of the star), jet flows from the disk corona (MHD active upper layer) of the magnetized rotating wind, and so on.

It is quite probable that in the internal regions of the protoplanetary disk (for small values of ϖ) at the early stage of its formation and in its upper layers (at large z there exist chaotic magnetic fields generated by a turbulent dynamo mechanism or just introduced into the disk with accreted interstellar plasma. These fields (whose energy could be comparable with the energy of hydrodynamic turbulence), mixed due to the differential rotation of the weakly ionized matter of the disk which experiences reconnection at its boundary, made a considerable contribution to the turbulent viscosity both in the internal region of the disk and at the external layers of its corona, where a high matter ionization degree was reached. The efficiency of MHD turbulence as the dissipation mechanism essentially depends on magnetic reconnection. We recall that the reconnection of magnetic field force lines (which represents the fundamental physical process in the space plasma responsible for many manifestations of its activity) is possible only in the case of complex plasma motion when magnetic force lines with different directions can closely approach each other. In this case, near the approaching point of force lines with different magnetic field direction, a rather high electric current density is achieved. In this plasma, before the beginning of reconnection there exists some accumulated magnetic energy, and after that the so-called tearing instability begins to develop; this instability finally results in force lines reconnection and the transformation of the excessive energy of the magnetic field into the kinetic or thermal energy of the plasma (see [Kadomtsev, 1987]).

In the protoplanetary disk, a large-scale magnetic field (whose characteristic size considerably exceeds the characteristic size of turbulent pulsations and is comparable with the size of the proto-Sun) is present in the protoplanetary disk along with the chaotic magnetic field; due to turbulent transfer, this field may be extended at least to the internal edge of the disk. This field penetrates into some region at both sides of the disk surface. In this case, the external region is influenced by magnetic stress caused by both small-scale field perturbations related with the turbulence in the disk and large-scale shear flow. As a result, not only effective turbulent viscosity and turbulent magnetic diffusion, but all of the effects related with the electrodynamics of average fields occur (see, e.g., [Zeldovich et al., 2006]). In particular, since the effective magnetic diffusion in the rotating conducting medium is necessarily accompanied by turbulent electromotive force $\alpha\mathbf{B}$ (the so-called α-effect related in the end with the influence of kinematical and magnetic helicity on the generation of the induced magnetic field, see [Moffatt 1980; Kolesnichenko, Marov, 2007]), it should be expected that the turbulent dynamo mechanism strongly influences the structure and evolution of the “young” proto planetary disk. It is known [Parker, 1955] that the small-scale reflection–non invariant (gyrotropic) turbulence in a rotating disk creates “loops” (the α-effect) when any magnetic field tube under the action of the local helical motion acquires the form of a twisted Ω symbol. This magnetic loop is accompanied by a current which is antiparallel (parallel) with respect to the applied average magnetic field component for right-skew (left-skew) random helical motions. The energy of Joule heat produced by these currents is a powerful source of heating for which, in particular, the disk corona with a thickness of an order of the disk thickness [Heyvaerts, Priest, 1992; Inverauity et al., 1995] is created. In reality, the corona may be much thicker, in spite of the fact that the “primary loops” floating to the surface in the turbulent medium under the action of the lifting force have this characteristic size. This is related with the fact that in the course of the reconnection of small loops, loops with a larger size may be formed [Galeev et al., 1979]. The corona keeps the magnetic connection of the remote regions of-the disk via large-scale force lines going through it, which are reconnected in the disk. Such a magnetic connection is a possible additional source of stress in the corona and, thus, its heating. Thus, because of viscous stress occurring due to the differential rotation of the magnetized accretion disk and the action of the turbulent dynamo, its corona is heated similar to the heating of the solar corona. The hot corona is capable of generating jet matter and field outflow. Actually, such a jet is a magnetized plasma wind outflowing the accreting disk (see, e.g., [Pudritz, Norman, 1986; Campbell, 2005]). In turn, the rotating wind transfers to infinity, together with matter and the magnetic field, the considerable angular momentum of the disk, thus allowing the disk to compress slowly and providing, along with the viscous transfer of the angular momentum outside, another possibility of removing the angular momentum from the disk [Konigl, Pudritz, 2000]. Note that the magnetic stress in the wind may also cause the very efficient focusing of the matter motion, jets (e.g., [Wang et al., 1990]).

As applied to the problem of the reconstruction of the evolution of the preplanetary gas–dust accretion disk, we developed in a number of papers [Kolesnichenko 2000, 2001, 2003-2005; Kolesnichenko, Marov, 2006-2008; Marov, Kolesnichenko, 2002, 2006] an approach to the solution of the problem of an adequate mathematical simulation of the turbulized disk medium taking into account the combined influence of magnetohydrodynamic effects and the effects of hydrodynamic turbulence on the-dynamics and processes of heat and mass transport in the differentially rotating space gas–dust plasma, the inertial properties of the polydispersive admixture of solid particles, the processes of coagulation and radiation, and a number of additional effects occurring during turbulent plasma motion in a magnetic field.

In particular, in the paper [Kolesnichenko, Marov, 2008] in the framework of the basic cosmogony problem related with the reconstruction of the protoplanetary accretion disk surrounding the Sun at the early stages of its existence, a closed system of magneto hydrodynamic equations with the scale of average motion was obtained in the approximation of the single-fluid magnetic hydrodynamics; this system is designated for the simulation of shear and convective turbulent flows of the weakly ionized disk medium in the presence of the magnetic field. This system of equations was used for a number of schematic formulations and numerical solutions of special problems on the self-consistent simulation of the structure and evolution of the turbulized matter in the magnetized disk and the related magnetized corona and in the case of matter accretion from ambient space. Thus, for example, if there is an ordered magnetic field with a noticeable perpendicular component in the disk, both the angular momentum and the energy can be carried away via magnetized plasma flows moving perpendicular to the disk plane, which results in the radial redistribution of the angular momentum and the matter of the disk and the angular velocity of the disk rotation different from the Keplerian one. In turn, the magnetic stress created in the corona by the conducting disk due to the relative shear in the magnetic force line bases results in the heating of local corona regions (via the turbulent dissipation mechanism) and influences the disk dynamics. Thus, in the case of the strict formulation of the above problems, it is necessary to take into account the existing magnetic connection of the protoplanetary disk and its corona.

In this paper, which continues the cycle of papers on the subject, we consider mainly the following four aspects of the problem of construction of the model of the structure and evolution of the protoplanetary disk of the Sun:

(i) the formulation of the basic system of averaged MHD equations for developed turbulence designated for setting and numerically solving various problems on the mutually consistent simulation of the structure and evolution of the disk and related corona at the early stages of their existence;

(ii) the development of a new approach to the simulation of the turbulent transport coefficient in the conducting disk which provides an account of the effects of the influence of the large-scale magnetic field generated by the turbulent dynamo mechanism and convective heat transport on the turbulence development in the density-stratified layer with a finite thickness, and thus allows one to reject the Shakura–Sunyaev α- formalism widely used in astrophysical literature;

1. Original equations of the problem

Based on results obtained in the paper [Kolesnichenko, 2008], we first present the complete system of averaged magnetohydrodynamic equations for the developed turbulence, in the framework of which a number of key models will be developed for the reconstruction of the course of evolution and the structure of the protoplanetary accretion disk near the young Sun. Below, we use two symbols for averaged parameters of the problem: the bar above a quantity means the conventional probability theoretical averaging of this quantity $\mathcal{A}(\mathbf{x},t)$ over an ensemble of possible implementations (time and/or space), while a tilde above a quantity means the weighted Favre averaging [Favre, 1969] determined by the relation $\langle\mathcal{A}\rangle \equiv \overline{\rho\mathcal{A}}/\bar{\rho}$ (where $\mathcal{A} = \langle\mathcal{A}\rangle + \mathcal{A}'' = \bar{\mathcal{A}} + \mathcal{A}'$; $\mathcal{A}'$, $\mathcal{A}''$ are the corresponding turbulent pulsations; $\overline{\mathcal{A}'} = 0$, $\overline{\rho\mathcal{A}''} = 0$; the properties of weighted averaging used in this paper can be found in [Kolesnichenko, Marov, 1999]). In the inertial reference frame, the averaged hydrodynamic equations for the developed turbulent flow and the magnetic induction equations for average magnetic field $\overline{\mathbf{B}}(\mathbf{x},t)$ in the absolute Gaussian system take the following form:

$$\frac{\partial\bar{\rho}}{\partial t}+\nabla\cdot\left(\bar{\rho}\langle\mathbf{u}\rangle\right)=0\,, \tag{1}$$

$$\bar{\rho}\frac{d\langle\mathbf{u}\rangle}{dt}=-\nabla\,(\bar{p}+p^{M}_{turb})+\nabla\cdot\mathbf{R}^{K}+\frac{1}{c}\bar{\mathbf{j}}\times\bar{\mathbf{B}}-\bar{\rho}\nabla\Psi_{G}, \tag{2}$$

$$\bar{\rho}\frac{d\langle\mathcal{E}\rangle}{dt}=-\nabla\cdot(\bar{\mathbf{q}}_{rad}+\mathbf{q}^{turb})-(\bar{p}+p^{M}_{turb})\nabla\cdot\langle\mathbf{u}\rangle+\mathbf{R}^{\mathrm{K}}:\nabla\langle\mathbf{u}\rangle+$$

$$+\frac{1}{4\pi\mu_0}\mathbf{R}^{M}:\nabla\bar{\mathbf{B}}+\frac{1}{\sigma_{\mathrm{e}}}\left|\bar{\mathbf{j}}\right|^{2}, \tag{3}$$

$$\bar{\rho}\frac{d}{dt}\left(\frac{\bar{\mathbf{B}}}{\bar{\rho}}\right)=\left(\bar{\mathbf{B}}\cdot\nabla\right)\langle\mathbf{u}\rangle+\nabla\cdot\mathbf{R}^{M}+\nu_{M}\nabla^{2}\bar{\mathbf{B}},\quad \nabla\cdot\bar{\mathbf{B}}=0\,, \tag{4}$$

$$\bar{p}=\Re\bar{\rho}\langle T\rangle. \tag{5}$$

Here, $d/dt=\partial/\partial t+\langle\mathbf{u}\rangle\cdot\nabla$ is the substantial time derivative for the averaged continuum; $\bar{\rho}(\mathbf{x},t)$, $\langle\mathbf{u}\rangle(\mathbf{x},\mathrm{t}):=\overline{\rho\mathbf{u}}/\bar{\rho}$ are, respectively, the averaged density and average weighted hydrodynamic velocity of the space matter in the disk ($\rho=\bar{\rho}+\rho'$; $\mathbf{u}(\mathbf{x},t)=\langle\mathbf{u}\rangle+\mathbf{u}''$; $\mathbf{u}''$is the Favre-averaged turbulent velocity pulsation); $\bar{\mathbf{B}}(\mathbf{x},t)$ is the averaged vector of the pulsating magnetic field (average magnetic field)[1)] strength;

$$\mathbf{R}^{M}(\mathbf{x},t):=-\left(\overline{\mathbf{u}''\mathbf{B}}-\overline{\mathbf{B}\mathbf{u}''}\right) \tag{6}$$

is the so called Reynolds magnetic tensor; $\nu_M:=c^2/4\pi\mu_0\sigma_{\mathrm{e}}$ is the molecular magnetic viscosity coefficient; c is the light velocity; μ_0 is the magnetic permeability; σ_e is the specific molecular electric conductance coefficient (it will be assumed below that μ_0,ν_M and σ_e are constant); $\bar{p}(\mathbf{x},t)$, $p^{M}_{turb}(\mathbf{x},t):=\overline{|\mathbf{B}'|^{2}}/8\pi\mu_0$ are, respectively, the average gas-dynamic pressure and the turbulent magnetic pressure;

$$\mathbf{R}^{K}(\mathbf{x},t):=\left\{-\overline{\rho\mathbf{u}''\mathbf{u}''}+\overline{\mathbf{B}'\mathbf{B}'}/4\pi\mu_0\right\}:=\mathbf{R}(\mathbf{x},t)+\boldsymbol{\tau}^{M}_{turb}(\mathbf{x},t) \tag{7}$$

[1)] In this paper, we do not distinguish between the magnetic field and magnetic induction, since the magnetic permeability of the disk medium is practically equal to unity.

is the Reynolds tensor of kinetic turbulent stress for the turbulized medium in the presence of the pulsating magnetic field; $\mathbf{R}(\mathbf{x},t) := -\overline{\rho \mathbf{u}''\mathbf{u}''}$ is the common Reynolds tensor for the gas which has the meaning of additional (turbulent) stress; $\boldsymbol{\tau}^{M}_{turb}(\mathbf{x},t) := \overline{\mathbf{B}'\mathbf{B}'} / 4\pi\mu_0$ is the magnetic stress tensor for the pulsating component of the magnetic field; $\bar{\mathbf{j}}(\mathbf{x},\mathrm{t})$ is the averaged conduction current (current measured by the observer moving together with the electro conductive gas) density included in the averaged Ampere's law,

$$\bar{\mathbf{j}} = (c / 4\pi\mu_0)\nabla \times \overline{\mathbf{B}}; \tag{8}$$

$$\Psi_{\mathrm{G}} := -G\mathcal{M}_{\odot}\left|\mathbf{x}\right|^{-1/2} \tag{9}$$

is the potential function of the gravitational field; $\mathcal{M}_{\odot}$ is the mass of the proto-Sun; G is the gravitational constant (below, we neglect the disk self-gravitation, which is always possible if $\mathcal{M}_{disk} / \mathcal{M}_{\odot} \le h / R_{\infty} << 1$; here $h(r)$ and R_{∞} are the half thickness and the external radius of the disk, respectively); $\langle\mathcal{E}\rangle(\mathbf{x},t) := \overline{\rho\mathcal{E}} / \bar{\rho}$ is the Favre-averaged specific internal energy $\mathcal{E}(\mathbf{x},\mathrm{t})$ of the disk medium (below, the internal energy of the gas is assumed to be proportional to temperature)

$$\mathcal{E}(\mathbf{x},t) := c_V T = \Re T(\gamma - 1)^{-1}, \tag{10}$$

where $\Re := \mathsf{R} / \mu$; R is the gas constant; μ is the average atomic mass (average mass per particle in units of $\mathrm{m_p}$); $\gamma := c_P / c_V$ is the adiabatic index; c_P, $c_V = \Re/(\gamma - 1)$ are, respectively, the specific heat capacity of the gas at constant pressure and constant volume (below, these quantities are assumed to be constant); $\bar{\mathbf{q}}_{rad}(\mathbf{x},t)$ is the averaged energy flux transferred by radiation; $\mathbf{q}^{turb}_{*}(\mathbf{x},t) \cong c_P \overline{\rho T''\mathbf{u}''}$ is the turbulent heat flux; and $\mathbf{q}^{turb}(\mathbf{x},t) := \mathbf{q}^{turb}_{*} - \overline{\mathrm{p}'\mathbf{u}''}$ is the reduced heat flux [Kolesnichenko, Marov, 1999].

The following should be noted in relation with the presented MHD equations for the averaged motion of the turbulized plasma. Upon the derivation of these equations, we did not take into account the radiation pressure and energy for simplicity, although sometimes it is necessary to consider the disk medium as a mixture of the ideal gas and the blackbody radiation. The generalization of Eqs. (2), (3), and (4) to this case does not present any difficulties. In averaged induction equation (4), the following term is included:

$$\nabla\cdot\mathbf{R}^{M}=\overline{\left(\mathbf{B}\cdot\nabla\right)\mathbf{u}''}-\overline{\left(\mathbf{u}''\cdot\nabla\right)\mathbf{B}}-\overline{\mathbf{B}\left(\nabla\cdot\mathbf{u}''\right)}=\nabla\times\left(\overline{\mathbf{u}''\times\mathbf{B}}\right):=c\nabla\times\boldsymbol{\mathcal{G}} \quad (11)$$

which plays the role of the additional source generating average magnetic field $\bar{\mathbf{B}}(\mathbf{x},t)$. Here,

$$\boldsymbol{\mathcal{G}}(\mathbf{x},\mathrm{t}):=c^{-1}\overline{\mathbf{u}''\times\mathbf{B}}=c^{-1}\overline{\rho\,\mathbf{u}''\times(\mathbf{B}/\rho)''}, \quad (12)$$

(or in tensor form, $\mathcal{G}_i:=-\varepsilon_{ijk}\mathrm{R}^{M}_{jk}/2c$) is the additional electromotive force generated by the random velocity and magnetic field fluctuations which appears in the averaged Ohm's law

$$\bar{\mathbf{j}}(\mathbf{x},t)=\sigma_e(\bar{\mathbf{E}}^{*}+\boldsymbol{\mathcal{G}}),\quad \bar{\mathbf{E}}^{*}(\mathbf{x},t):=\bar{\mathbf{E}}+c^{-1}\langle\mathbf{u}\rangle\times\bar{\mathbf{B}}; \quad (13)$$

$\bar{\mathbf{E}}(\mathbf{x},t)$ is the averaged electric field strength vector; and ε_{ijk} is the completely antisymmetric third rank tensor (alternating Levi–Civita tensor). Note that one of the main objectives of the semi empirical theory of MHD turbulence is the construction of a special closing relation for turbulent flux $\boldsymbol{\mathcal{G}}(\mathbf{x},t)$ as a function of average fields $\bar{\mathbf{B}}(\mathbf{x},t)$ and $\langle\mathbf{u}\rangle(\mathbf{x},t)$ so that it would be possible to find $\bar{\mathbf{B}}(\mathbf{x},t)$ from induction equation (4) for the given field $\langle\mathbf{u}\rangle(\mathbf{x},t)$. Taking into account (12), the last term for the averaged motion in heat inflow equation (3) can be represented in the form

$$(1/4\pi\mu_0)\mathbf{R}^{M}:\nabla\bar{\mathbf{B}}=-\boldsymbol{\mathcal{G}}\cdot\bar{\mathbf{j}}\ . \quad (14)$$

Finally, it is important to note that the substantial internal energy balance equation takes form (3) only in the case of the special regime of the strongly developed turbulencein the system, when in the structure of the pulsating fields $\mathbf{u}''(\mathbf{x},t)$ and $\mathbf{B}'(\mathbf{x},t)$ a quasi-stationary state is established, such that the total turbulent plasma energy $\langle b\rangle_\Sigma(\mathbf{x},t)$ equal to the sum of turbulent gas energy $\langle b\rangle(\mathbf{x},t):=\overline{\rho\left|\mathbf{u}''\right|^2}/2\bar{\rho}$ and $\langle b\rangle_M(\mathbf{x},t):=\overline{\left|\mathbf{B}'\right|^2}/8\pi\mu_0\bar{\rho}$ – turbulent magnetic field energy

$$\langle b\rangle_\Sigma:=\langle b\rangle+\langle b\rangle_M=\overline{\rho\left|\mathbf{u}''\right|^2}/2\bar{\rho}+\overline{\left|\mathbf{B}'\right|^2}/8\pi\mu_0\bar{\rho}, \quad (15)$$

slightly varies in time and space, $d\langle b\rangle_\Sigma/dt\cong 0$ [Kolesnichenko, Marov, 2008]. System of equations (1)-(5) should be added by the defining relations for turbulent fluxes and the expressions for the necessary thermodynamic characteristics and transport. The boundary and initial conditions for the structural parameters do not differ from the corresponding conditions for non electro conductive media, but it is necessary to use additional conditions for the average magnetic field.

Total averaged energy conservation law. Further, we will need the balance equation for the total averaged energy of the disk system equal to the sum $\langle \mathrm{U} \rangle_{tot} = \langle \mathrm{U} \rangle_{tot}^{sub} + \langle \mathcal{E} \rangle^{M}$ of the Favre-averaged total energy of the conducting gas

$$\langle \mathrm{U} \rangle_{tot}^{sub}(\mathbf{x},t) := \langle \mathcal{E} \rangle + \Psi_G + \left| \langle \mathbf{u} \rangle \right|^2 / 2 + \langle b \rangle \tag{16}$$

and the average energy of the electromagnetic field

$$\langle \mathcal{E} \rangle_M(\mathbf{x},t) := \overline{\rho \mathcal{E}_M} / \overline{\rho} = \overline{\rho(\left|\mathbf{B}\right|^2 / 8\pi\mu_0\rho)} / \overline{\rho} = \left|\overline{\mathbf{B}}\right|^2 / 8\pi\mu_0\overline{\rho} + \langle b \rangle_M \tag{17}$$

According to [Kolesnichenko, Marov, 2008], this equation for a developed turbulence can be written in the form

$$\frac{\partial}{\partial t}\left(\overline{\rho}\langle \mathrm{U} \rangle_{tot}^{sub} + \overline{\rho}\langle \mathcal{E} \rangle^{M}\right) = \mathrm{Q}_{rad} -$$

$$-\nabla \cdot \left\{ \left(\overline{\rho}\langle \mathrm{U} \rangle_{tot}^{sub}\right)\langle \mathbf{u} \rangle + \mathbf{q}^{turb} + \overline{p}\langle \mathbf{u} \rangle - \left(\frac{\left|\overline{\mathbf{B}}\right|^2}{8\pi\mu_0} \right)\langle \mathbf{u} \rangle + \overline{\mathbf{q}_{Poynt}} - \mathbf{R} \cdot \langle \mathbf{u} \rangle + \overline{\rho b'' \mathbf{u}''} \right\}, \tag{18}$$

where

$$\overline{\mathbf{q}_{Poynt}} := c\, \overline{\mathbf{E} \times \mathbf{B}} / 4\pi \tag{19}$$

is the averaged Umov–Poynting vector which has the meaning of the energy flux density of the electromagnetic field,

$$\mathrm{Q}_{rad} := -\nabla \cdot \overline{\mathbf{q}}_{rad} = \mathcal{A} - \mathcal{B} = \int_0^\infty \int_\Omega \rho \kappa_{\nu a} I_\nu d\mathbf{\Omega}\, d\nu - 4\pi \int_0^\infty \rho \kappa_{\nu a} \mathrm{B}_\nu d\nu , \tag{20}$$

where ν, $I_\nu(\mathbf{x},\mathbf{\Omega},t)$ and $\mathrm{B}_\nu(\mathbf{x},\mathbf{\Omega},t)$, respectively, are the radiation frequency, spectral intensity, and internal source function; $\mathbf{\Omega}$ is the direction of motion of photons; and $\kappa_{\nu a}$ is the true radiation absorption coefficient for the disk matter (spectral opacity). The first ($\mathcal{A}$) term in expression (20) corresponds to the absorbed, and the second ($\mathcal{B}$) term, to the spontaneously emitted in unit volume and unit time radiation energy. Several radiation transport regimes are possible which are applicable in different regions of the disk and (depending on the accretion rate, protostar mass, etc.) different models of the disk. In particular, if the total optical thickness of the disk $d\tau_\nu = \rho\kappa_{\nu a} ds$ along the direction of propagation *s* exceeds unity, photons are transferred to the disk surface via diffusion (see relation (26)). In the general case, the spectral intensity $I_\nu(\mathbf{x},\boldsymbol{\Omega},t)$ included in formula (20) should be determined in the course of the solution of the radiation transport equation.

In the MHD approximation, vector can be transformed to the form [Kolesnichenko, Marov, 2008]

$$\overline{\mathbf{q}_{Poynt}} = \bar{\rho}\left(\frac{|\bar{\mathbf{B}}|^2}{8\pi\mu_0\bar{\rho}} + b_{\mathrm{M}} + \frac{p^M_{turb}}{\bar{\rho}}\right)\langle\mathbf{u}\rangle + \left(\frac{|\bar{\mathbf{B}}|^2}{8\pi\mu_0}\mathbf{I} - \frac{\bar{\mathbf{B}}\bar{\mathbf{B}}}{4\pi\mu_0}\right)\cdot\langle\mathbf{u}\rangle - \boldsymbol{\tau}^M_{turb}\cdot\langle\mathbf{u}\rangle +$$

$$+\frac{1}{4\pi\mu_0}\overline{(|\mathbf{B}|^2\,\mathbf{I} - \mathbf{BB})\cdot\mathbf{u}''} - \nu_M\nabla\cdot\left(\frac{|\bar{\mathbf{B}}|^2}{8\pi\mu_0}\mathbf{I} - \frac{\bar{\mathbf{B}}\bar{\mathbf{B}}}{4\pi\mu_0}\right) - \nu_M\nabla\cdot\left(p^M_{turb}\mathbf{I} - \boldsymbol{\tau}^M_{turb}\right). \quad (19^*)$$

Note that for strongly developed turbulence, two small terms in this expression, including “molecular” magnetic viscosity coefficient ν_M, can be omitted for most spatial regions of the accretion disk and the corona (see, e.g., [Lazarian, Vishniac, 1999]). These terms should be taken into account only in regions with high spatial gradients of the magnetic field, for example, in the region of stochastic reconnection of magnetic force lines.

Combining (18) and (19*), we write the conservation law for the total energy of the disk system in the following form:

$$\frac{d}{dt}(\bar{\rho}\langle \mathrm{U}\rangle_{tot}) = \mathrm{Q}_{rad} - \nabla\cdot\left\{\mathbf{q}^{turb} + (\bar{p} + p^M_{turb})\langle\mathbf{u}\rangle - \left(\mathbf{R}^K + \frac{\bar{\mathbf{B}}\bar{\mathbf{B}}}{4\pi\mu_0}\right)\cdot\langle\mathbf{u}\rangle + \right.$$

$$\left. + \overline{\rho\, b''\mathbf{u}''} + \frac{1}{4\pi\mu_0}\overline{\rho\left(\frac{|\mathbf{B}|^2\mathbf{I} - \mathbf{B}\mathbf{B}}{\rho}\right)''\cdot\mathbf{u}''}\right\}, \quad (18^*)$$

and the two correlation terms on the left-hand side of this equation can be neglected due to their smallness in the considered problem (see, e.g., [Pudritz, 1981]).

2. Stationary nonequilibrium regime of the subsystem of turbulent chaos. Derivation of the defining relations

In the paper [Kolesnichenko, Marov, 2008], the turbulent motion of the electroconductive gas was described in the framework of a two-fluid thermodynamic continuum consisting of two mutually open subsystems continuously filling the same coordinate space: the subsystem of the average motion and the subsystem of turbulent chaos connected with pulsation motion of the matter and field. It was assumed that elementary volume $d\mathbf{x}$ of the subsystem of turbulent chaos can be characterized by the generalized thermodynamic state parameters, such as entropy $S_{turb}(\mathbf{x},t)$, internal energy $\mathcal{E}_{turb}(\mathbf{x},t)$, pressure $p_{turb}(\mathbf{x},t)$, and turbulization temperature $T_{turb}(\mathbf{x},t)$ (the quantity characterizing the degree of intensity of turbulent pulsations [Blackadar, 1955]). Entropy $S_{turb}(\mathbf{x},t)$ and internal energy $\mathcal{E}_{turb}(\mathbf{x},t)$ of turbulization were considered as the primary concepts and were introduced a priori for providing the consistency of the thermodynamic theory; their exact physical interpretation was not assumed [Jou et al. 2006]. The mentioned quasi-equilibrium regime of motion in the subsystem of turbulent chaos was specially analyzed; in this regime, total occurrence $\sigma_{(S_{turb})} \equiv \sigma^{e}_{(S_{turb})} + \sigma^{i}_{(S_{\mathrm{turb}})}$ of turbulization entropy $S_{turb}(\mathbf{x},t)$ is almost absent. This condition means that occurrence $\sigma^{i}_{(S_{turb})}(\mathbf{x},t)$ of entropy $S_{turb}(\mathbf{x},t)$ (due to irreversible processes inside the subsystem of turbulent chaos) is compensated by its outflow $\sigma^{e}_{(S_{turb})}(\mathbf{x},t)$ into the “external medium” (i.e., to the subsystem of averaged motion) in such a degree that $\sigma_{(S_{turb})}(\mathbf{x},t) \cong 0$. Since the following inequality is always satisfied, $\sigma^{i}_{(S_{turb})}(\mathbf{x},t) \geq 0$, the following expression is valid $0 > \sigma^{e}_{(S_{turb})} \cong -\sigma^{i}_{(S_{turb})}$. This yields that for preserving such a stationary–nonequilibrium turbulence regime, the inflow of negative entropy (negentropy) from the averaged motion to the chaotic component is necessary, $\sigma^{e}_{(S_{turb})} = -\mathrm{T}\sigma^{e}_{\langle S\rangle} / \mathrm{T}_{\mathrm{turb}} < 0$ [Kolesnichenko, 2003]. Only in this case, the balance equation for averaged entropy of the system $\langle S\rangle(\mathbf{x},t) = c_V \ln\left(\langle T\rangle / \overline{\rho}^{\,\gamma-1}\right)$ takes the “standard” form of the general heat transport equation [Marov, Kolesnichenko, 2006],

$$\bar{\rho}\frac{d\langle S\rangle}{dt}+\nabla\cdot\left(\mathbf{q}^{turb}/\langle T\rangle\right)=\sigma_{\langle S\rangle}\ , \tag{21}$$

where local occurrence $\sigma_{\langle S\rangle}(\mathbf{x},t)$ of entropy $\langle S\rangle(\mathbf{x},t)$ due to dissipative processes in the electroconductive turbulized medium is determined by the expression

$$0\le\langle T\rangle\,\sigma_{\langle S\rangle}(\mathbf{x},t)=$$
$$=-\mathbf{q}^{turb}\cdot\nabla\ln\langle T\rangle+\left[\mathbf{R}^{K}+\tfrac{2}{3}\bar{\rho}\left(\langle b\rangle-\langle b\rangle_{\mathrm{M}}\right)\mathbf{I}\right]:\overset{0}{\boldsymbol{\mathcal{D}}}+\frac{\mathbf{R}^{M}}{4\pi\mu_0}:\left(\nabla\bar{\mathbf{B}}\right)^{a}+\frac{\left|\bar{\mathbf{j}}\right|^{2}}{\sigma_e}+\mathrm{Q}_{rad}. \tag{22}$$

Here $\overset{0}{\boldsymbol{\mathcal{D}}}:=\left(\partial\langle\mathbf{u}\rangle/\partial\mathbf{x}\right)^{s}-\tfrac{1}{3}\mathbf{I}\left(\partial/\partial\mathbf{x}\right)\cdot\langle\mathbf{u}\rangle$ is the shear velocity for the averaged motion; $\mathbf{I}$ is the unit tensor; $\left(\partial\langle\mathbf{u}\rangle/\partial\mathbf{x}\right)^{s}_{jk}:=\tfrac{1}{2}\left(\partial\langle u\rangle_j/\partial x_k+\partial\langle u\rangle_k/\partial x_j\right)$ and $\left(\partial\bar{\mathbf{B}}/\partial\mathbf{x}\right)^{a}_{jk}:=\tfrac{1}{2}\left(\partial\bar{B}_j/\partial x_k-\partial\bar{B}_k/\partial x_j\right)$ are, respectively, the symmetric and anti-symmetric parts of tensors $\nabla\langle\mathbf{u}\rangle$ and $\nabla\bar{\mathbf{B}}$.

If the Onsager non equilibrium thermodynamics method is used, bilinear form (22) for $\sigma_{\langle S\rangle}(\mathbf{x},t)$ provides the defining relations for the turbulent heat flux $\mathbf{q}^{turb}(\mathbf{x},t)$, total turbulent stress tensor $\mathbf{R}^{K}(\mathbf{x},t)$, and magnetic Reynolds tensor $\mathbf{R}^{M}(\mathbf{x},t)$ corresponding to the regime of the stationary–nonequilibrium state of the turbulent field. For isotropic turbulence (in this paper, we consider this case only), if the Curie–Prigozhin principle is used(according to which the connection between the tensors of different rank in an isotropic medium is impossible), these relations take the following form (the small cross terms being neglected) [Marov, Kolesnichenko, 2002]:

$$\mathbf{q}^{turb}(\mathbf{x},t)=-\lambda^{turb}\left(\nabla\langle T\rangle-\frac{\nabla\bar{p}}{\bar{\rho}c_P}\right)=-\lambda^{turb}\frac{\langle T\rangle}{c_P}\nabla\langle \mathrm{S}\rangle\cong-\lambda^{turb}\left(\nabla\langle T\rangle-\frac{\mathbf{g}}{c_P}\right), \tag{23}$$

$$\mathbf{R}^{K}(\mathbf{x},t)=-\tfrac{2}{3}\bar{\rho}\left(\langle b\rangle-\langle b\rangle_{M}\right)\mathbf{I}+2\bar{\rho}\nu_K^{turb}\left\{\left(\nabla\langle\mathbf{u}\rangle\right)^{\mathrm{s}}-\tfrac{1}{3}\mathbf{I}\left(\nabla\cdot\langle\mathbf{u}\rangle\right)\right\}, \tag{24}$$

$$\mathbf{R}^{M}(\mathbf{x},t)=2\nu_{M}^{turb}\left(\nabla\bar{\mathbf{B}}\right)^{a}, \qquad \left(or\ c\boldsymbol{\mathcal{G}}=-\nu_{M}^{turb}\nabla\times\bar{\mathbf{B}}\right), \tag{25}$$

where $\lambda^{turb}(\mathbf{x},t)$, $\nu_{K}^{turb}(\mathbf{x},t)$ and $\nu_{M}^{turb}(\mathbf{x},t)$ are, respectively, the coefficients of turbulent heat conduction, turbulent kinematical viscosity, and turbulent magnetic field diffusion depending in the general case on the following parameters: $\bar{\rho}$, $\nabla\langle\mathbf{u}\rangle$, $\bar{\mathbf{B}}/4\pi\mu_0$ and L (here, $L(\mathbf{x})$ is some geometric characteristic of the position of point $\mathbf{x}$, for example, equal to the common "shear path length" [Ievlev, 1975]).

Below, the defining relation for the radiation vector will be used in the form of the radiant heat flux,

$$\mathbf{q}_{rad}(\mathbf{x},t)=-\chi_r\nabla\langle T\rangle=-\frac{16\sigma_{\mathrm{B}}\langle T\rangle^3}{3\kappa\bar{\rho}}\nabla\langle T\rangle\ . \tag{26}$$

This formula is valid in the case of the diffusion of equilibrium radiation (for example, in the case of the local thermodynamic equilibrium of radiation and matter inside an optically thick disk). Here, σ_{B}, $\chi_r=16\sigma_B\langle T\rangle^3/3\kappa\bar{\rho}$ are, respectively, the Stefan–Boltzmann constant and the radiant (nonlinear) heat conduction coefficient of the medium which strongly depends on the material temperature and density; $\kappa(\rho,T)$ is the total opacity of the medium which depends on ρ and T in a complex way, as well as on the ionization degree, chemical composition, etc. [Fridman, Bisikalo, 2008]. In the general case, κ is determined as the Rosseland mean with respect to inverse spectral opacities $1/\kappa_\nu$ (see, e.g., [Frank-Kamenetsky, 1959]). It is known that the dominating contribution of κ_{ff} to opacity κ in the accretion disk is introduced by the nonrelativistic thermal bremsstrahlung, or "free–free" transitions. Below, the absorbed Rosseland mean opacity κ related with these processes is determined by the Kramers formula

$$\kappa_{ff}(\rho,T)=\mathrm{K}\,\rho\,T^{-7/2}\ \mathrm{cm}^2\ \mathrm{g}^{-1}, \tag{27}$$

where $\mathrm{K}=0.32\times10^{23}$ is the constant. In optically thick disks, the comparable (but smaller) quantity $\kappa_{es}=2\times10^{-2}(1+\mathrm{X})\,\mathrm{cm}^2\ \mathrm{g}^{-1}$ is introduced by "bound–bound" transitions in lines and "bound– free" ionization transitions (where X is the mass fraction of hydrogen in the medium).

Let us make an important remark concerning formula (25) for electromotive force $\mathcal{G}(\mathbf{x},t)$. This formula is valid only for an isotropic (in the hydrodynamic sense) turbulence when pulsating velocity field $\mathbf{u}''(\mathbf{x},t)$ possesses mirror symmetry in the whole system. However, in the case of a rotating accretion disk, it is possible that, for example, in the upper part of the disk left-rotating turbulent motion is more probable than the right-rotating one, or vice versa.

The physical reason of the violation of the reflective symmetry is the influence of the Coriolis force on vortices floating up and down in the turbulent medium of the disk. In this case, the mirror symmetry of field $\mathbf{u}''(\mathbf{x},t)$ with respect to the central plane of the disk is absent and the turbulence can possess the so-called hydrodynamic helicity density $h_{hel}(\mathbf{x},t) := \overline{\mathbf{u}'' \cdot (\nabla \times \mathbf{u}'')}$, which characterizes the excess of vortices of a given sign [Moffatt, 1980; Vainshtein et al., 1980; Krause, Radler 1984; Kolesnichenko, Marov 2007]. The generalization of formula (25) to the case of mirror-nonsymmetric turbulence takes the form (see, e.g., [Steenbeck et al., 1966]),

$$c\mathcal{G}(\mathbf{x},t) = \alpha \overline{\mathbf{B}} - \nu_M^{turb} \nabla \times \overline{\mathbf{B}} \tag{25*}$$

where helicity coefficient α is pseudoscalar. It can be easily seen that the additional term in relation (25*) is connected with electric current $\overline{\mathbf{j}}(\mathbf{x},t) = \sigma_e \alpha \overline{\mathbf{B}} + ...$ directed along the magnetic field. Simple considerations show that for the case of isotropic and mirror-symmetric velocity field $\mathbf{u}''(\mathbf{x},t)$, helicity coefficient α is equal to zero. Indeed, for an isotropic medium the probability of some given implementation of the ensemble of this field and the implementation obtained from it by mirror reflection is the same. Then, on one hand, α should not change if this reflection is performed, since the ensemble has not changed, but on the other hand, α should change sign, since it is a pseudoscalar; therefore, $\alpha = 0$.

Substituting (25*) into induction equation (4) for average fields, we obtain

$$\overline{\rho}\frac{d}{dt}\left(\frac{\overline{\mathbf{B}}}{\overline{\rho}}\right) = \left(\overline{\mathbf{B}} \cdot \nabla\right)\langle \mathbf{u} \rangle + \left(\nu_M + \nu_M^{turb}\right)\nabla^2 \overline{\mathbf{B}} - \nu_M^{turb} \nabla \times \left\{\nabla \times \overline{\mathbf{B}}\right\} + \alpha \nabla \times \overline{\mathbf{B}}. \tag{28}$$

For a well mixed turbulence (created by a $\mathbf{u}''(\mathbf{x},t)$' field) when the magnetic field becomes tangled and small scale, the process of diffusion is enhanced, $\nu_M^{turb} >> \nu_M > 0$ (condition of strongly developed turbulence). Below, it will be assumed for simplicity that in Eq. (28) ν_M^{turb} and α are constant; then

$$\bar{\rho}\frac{d}{dt}\left(\frac{\bar{\mathbf{B}}}{\bar{\rho}}\right)=\left(\bar{\mathbf{B}}\cdot\nabla\right)\langle\mathbf{u}\rangle+\nu_M^{turb}\nabla^2\bar{\mathbf{B}}+\alpha\nabla\times\bar{\mathbf{B}}. \qquad (28^*)$$

Then, it can be seen that the reflection–symmetric isotropic turbulence, unlike the gyrotropic one, causes only turbulent magnetic field diffusion.

It should be noted that due to the pseudoscalar nature, the effect is antisymmetric with respect to the central plane of the disk. The symmetry properties of Maxwell equations admit two types of symmetry for eigensolutions (modes) of average field dynamo equation (28*): magnetic fields can be antisymmetric with respect to the equator (dipole symmetry) and symmetric with respect to the equator (quadruple symmetry). In particular, the solar dynamo mechanism, as a rule, excites mainly the dipole oscillating mode (the Hale rule).

If expressions (25*) and (8) are substituted into Ohm's law (13), we obtain for the averaged current

$$\bar{\mathbf{j}}(\mathbf{x},t)=\sigma_{\mathrm{e}}^{turb}\bar{\mathbf{E}}^*+\frac{c\alpha}{4\pi\mu_0\nu_M^{turb}}\bar{\mathbf{B}}\cong\sigma_{\mathrm{e}}^{turb}\left(\bar{\mathbf{E}}^*+\alpha c^{-1}\bar{\mathbf{B}}\right). \qquad (29)$$

Here, turbulent conductivity $\sigma_{\mathrm{e}}^{turb}$ is determined by the formula

$$\sigma_{\mathrm{e}}^{turb}=\frac{\sigma_e}{1+4\pi\mu\nu_M^{turb}\sigma_e/c^2}=\frac{\sigma_e\nu_M}{\nu_M+\nu_M^{turb}}\cong\frac{\sigma_e\nu_M}{\nu_M^{turb}}=\frac{c^2}{4\pi\mu_0\nu_M^{turb}}\ , \qquad (30)$$

it can be seen from this formula that turbulent conductivity $\sigma_{\mathrm{e}}^{turb}$ in the case of developed turbulence is smaller than molecular conductivity σ_e.

3. Derivation of the correction function to the turbulent viscosity coefficient for a conducting medium with a variable density

General heat transport equation (21) with accounting for defining relations (23)-(25) acquires the form

$$\langle T\rangle\bar{\rho}\frac{d\langle S\rangle}{dt}-\nabla\cdot\left\{\lambda^{turb}\left(\nabla\langle T\rangle-\frac{1}{\bar{\rho}c_P}\nabla\bar{p}\right)\right\}=$$

$$= 2\bar{\rho}\nu_K^{turb}\left(\overset{0}{\boldsymbol{\mathcal{D}}}:\overset{0}{\boldsymbol{\mathcal{D}}}\right) + \frac{\nu_{\mathrm{M}}^{turb}}{2\pi\mu_0}\left(\nabla\bar{\mathbf{B}}\right)^a:\left(\nabla\bar{\mathbf{B}}\right)^a + \frac{\left|\bar{\mathbf{j}}\right|^2}{\sigma_e} + Q_{rad}. \qquad (22^*)$$

Since, due to (30) we have

$$\frac{\nu_M^{turb}}{2\pi\mu_0}\left(\nabla\bar{\mathbf{B}}\right)^a:\left(\nabla\bar{\mathbf{B}}\right)^a + \frac{\left|\bar{\mathbf{j}}\right|^2}{\sigma_e} = \frac{\left|\bar{\mathbf{j}}\right|^2}{\sigma_e^{turb}} + \frac{\left|\bar{\mathbf{j}}\right|^2}{\sigma_e} = \left|\bar{\mathbf{j}}\right|^2\frac{\sigma_e + \sigma_e^{turb}}{\sigma_e^{turb}\sigma_e} \cong \frac{\left|\bar{\mathbf{j}}\right|^2}{\sigma_e^{turb}},$$

equation (22*) can be rewritten in the following final form:

$$\langle T\rangle\bar{\rho}\frac{d\langle S\rangle}{dt} \cong \nabla\cdot\left(\frac{\lambda^{turb}\langle T\rangle}{c_P}\nabla\langle S\rangle\right) + 2\bar{\rho}\nu_K^{turb}\left(\overset{0}{\mathbf{D}}:\overset{0}{\mathbf{D}}\right) + \frac{\left|\bar{\mathbf{j}}\right|^2}{\sigma_e^{turb}} + Q_{rad}, \qquad (22^{**})$$

where is the specific entropy of the system. Here, the quantity represents the amount of heat (per unit volume of the medium) received by the system in unit time, the first term on the right is the heat supplied to the considered volume via turbulent heat conduction, the second term represents the energy dissipated in the form of heat due to turbulent viscosity, the third term corresponding to Joule heating takes into account the contribution of the average magnetic field into the system entropy production, and finally, the last term $Q_{rad} := -\nabla\cdot\mathbf{q}_{rad}$ is related with the process of radiant heat release from the system.

It is known that in the case of isotropic turbulence, the coefficients of turbulent kinematical viscosity ν_K^{turb} and turbulent magnetic field diffusion ν_M^{turb} are close to the product $\mathrm{w}_{turb} l_{cor}$ of velocity of turbulent vortices $\mathrm{w}_{turb} \cong \sqrt{\left|\mathbf{u}''\right|^2}$ and their correlation length l_{cor} and helicity coefficient $\alpha \cong -\frac{1}{3}h_{hel}\tau_{cor}$, where

$$h_{hel} \equiv \overline{\mathbf{u}''\cdot\left(\nabla\times\mathbf{u}''\right)}$$

is the density of hydrodynamic helicity (pseudoscalar) and τcor is the scale characterizing the time variation of the turbulent velocity $\mathbf{u}''(\mathbf{x},t)$ field (see, e.g., [Krause, Radler, 1984]). In particular, if, according to the standard Shakura–Sunyaev hypothesis [Shakura, 1972], it is assumed that l_{cor} is the effective half width of the accretion disk and w_{turb} is expressed in terms of the thermal speed of sound c_s, turbulent diffusion results in the characteristic magnetic field damping time (or, more precisely, those of its components which change noticeably on the scale of the disk thickness) of an order of the period of Keplerian rotation. In this case, the magnetic Reynolds number $\mathrm{Re}_M \propto 1$ and turbulent transport is important.

At the same time, generalizing the known Kolmogorov formula for a nonconducting fluid to the case of MHD turbulence, it can be assumed that kinetic turbulent viscosity coefficient ν_K^{turb} is calculated using the formula

$$\nu_K^{turb} = L\sqrt{b_\Sigma}\,, \tag{31}$$

where L is the mixing length according to Prandtl (the numerical factor can be included in L). This assumption is often quite acceptable for practical applications. At the same time, the possible influence of the magnetic field on the character of mixing is not explicitly taken into account in relation (31), which is inadmissible for developed MHD turbulence (for example, for largescale perturbations). Therefore, in the general case it is necessary to introduce the correction taking into account the inverse effect of magnetic field diffusion and heat transport on the turbulence development in the electroconductive disk medium in formula (31).

For finding this correction factor to L, we use the balance equation for the turbulization entropy $S_{turb}(\mathbf{x},t)$, which in the case of the stationary–nonequilibrium regime of developed turbulence takes the form [Kolesnichenko, Marov 2008]

$$0 \cong \bar{\rho}\frac{dS_{turb}}{dt} + \nabla\cdot\mathbf{J}_{S_{turb}} =$$

$$= 2\bar{\rho}\nu_K^{turb}\left(\overset{0}{\boldsymbol{\mathcal{D}}}:\overset{0}{\boldsymbol{\mathcal{D}}}\right) + \frac{\nu_M^{turb}\left(\nabla\times\overline{\mathbf{B}}\right)^2}{4\pi\mu} + \frac{\lambda^{turb}\mathbf{g}}{c_p\langle T\rangle}\cdot\left(\nabla\langle T\rangle - \frac{\mathbf{g}}{c_p}\right) - \bar{\rho}\varepsilon_\Sigma. \tag{32}$$

Here, $\varepsilon_\Sigma := \langle\varepsilon_M\rangle + \langle\varepsilon_b\rangle = -\left\{\overline{\rho(\boldsymbol{\tau}/\rho)'' : \nabla\mathbf{u}''} + \left(\nu_M / 4\pi\mu_0\right)\overline{\left|\nabla\times\mathbf{B}'\right|^2}\right\}$ is the total specific dissipation rate of turbulent kinetic and turbulent magnetic energy into heat (under the action of molecular kinematical viscosity and magnetic field viscosity); $\Phi_\mathrm{v} \equiv 2\bar{\rho}\nu_K^{turb}\left(\overset{0}{\boldsymbol{\mathcal{D}}}:\overset{0}{\boldsymbol{\mathcal{D}}}\right)$ is the dissipative function, $\boldsymbol{\tau}$ is the viscous stress tensor related with the processes of molecular transport of the disk matter momentum, and $\mathbf{g} = -\nabla\Psi_G = G\mathcal{M}_\odot\mathbf{r}/\left|\mathbf{r}\right|^3$.

Using the notation w_{turb} for the characteristic pulsation velocity of the conducting medium and L for the Prandtl mixing length (in the case of absence of a magnetic field), we write

$$\nu_K^{turb} = L w_{turb}, \quad \nu_M^{turb} = \frac{L w_{turb}}{\mathrm{Pr}_M^{turb}}, \quad \frac{\lambda^{turb}}{\bar{\rho}\langle c_p \rangle} = \frac{L w_{turb}}{\mathrm{Pr}_K^{turb}}, \quad \varepsilon_\Sigma = \frac{1}{\alpha_{ss}^2} \frac{w_{turb}^3}{L}. \qquad (33)$$

In this case, empirical constant α_{ss} and turbulent numbers of Prandtl (kinetic and magnetic ones)

$$\mathrm{Pr}_K^{turb} = \bar{\rho} c_p \nu_K^{turb} / \lambda^{turb}, \quad \mathrm{Pr}_M^{turb} = \nu_K^{turb} / \nu_M^{turb}. \qquad (34)$$

Substituting these expressions into (32), we obtain for the stationary regime

$$w_{turb} \left\{ 2L \left(\overset{0}{\boldsymbol{\mathcal{D}}} : \overset{0}{\boldsymbol{\mathcal{D}}} \right) + \frac{L}{\mathrm{Pr}_M^{turb}} \frac{\left(\nabla \times \overline{\mathbf{B}} \right)^2}{4\pi\mu\bar{\rho}} + \frac{L}{\mathrm{Pr}_K^{turb}} \frac{\mathbf{g}}{\langle T \rangle} \cdot \left(\nabla \langle T \rangle - \frac{\mathbf{g}}{c_p} \right) - \frac{w_{turb}^2}{\alpha_{ss}^2 L} \right\} \approx 0$$

(35)

Equation (35) is separated into two equations: the equation $w_{turb} = 0$ corresponding to the laminar flow regime, and the equation

$$w_{turb}^2 = \alpha_{ss}^2 L^2 \left\{ 2 \left(\overset{0}{\boldsymbol{\mathcal{D}}} : \overset{0}{\boldsymbol{\mathcal{D}}} \right) + \frac{1}{\mathrm{Pr}_M^{turb}} \frac{\left(\nabla \times \overline{\mathbf{B}} \right)^2}{4\pi\mu_0\bar{\rho}} + \frac{1}{\mathrm{Pr}_K^{turb}} \frac{\mathbf{g}}{\langle T \rangle} \cdot \left(\nabla \langle T \rangle - \frac{\mathbf{g}}{c_p} \right) \right\} \qquad (36)$$

describing the established turbulent regime. Equation (36) has a real solution if

$$2 \left(\overset{0}{\boldsymbol{\mathcal{D}}} : \overset{0}{\boldsymbol{\mathcal{D}}} \right) + \frac{1}{\mathrm{Pr}_K^{turb}} \left\{ \frac{\left(\nabla \times \overline{\mathbf{B}} \right)^2}{4\pi\mu_0\bar{\rho}} + \frac{\mathbf{g}}{\langle T \rangle} \cdot \left(\nabla \langle T \rangle - \frac{\mathbf{g}}{c_p} \right) \right\} \geq 0$$

which yields $\mathrm{Ri}_\Sigma \equiv \mathrm{Ri}_K - \mathrm{Ri}_M \leq \left(\mathrm{Ri}_\Sigma \right)_{cr} = \mathrm{Pr}_K^{turb}$ where the following notation is introduced:

$$\mathrm{Ri}_K := -\frac{\mathbf{g}\cdot\left(\nabla\langle T\rangle - \mathbf{g}/c_p\right)/\langle T\rangle}{2\left(\overset{0}{\boldsymbol{\mathcal{D}}}:\overset{0}{\boldsymbol{\mathcal{D}}}\right)}, \quad \mathrm{Ri}_M \equiv \frac{1}{4\pi\mu_0\bar{\rho}}\frac{\left(\nabla\times\overline{\mathbf{B}}\right)^2}{2\left(\overset{0}{\boldsymbol{\mathcal{D}}}:\overset{0}{\boldsymbol{\mathcal{D}}}\right)}. \tag{37}$$

Here Ri_K and Ri_M are, respectively, the hydrodynamic Richardson number (a dimensionless quantity determining the relative contribution of thermal matter convection into turbulent energy generation, compared with energy transfer from averaged motion) and the gradient magnetohydrodynamic Richardson number (proportional to the ratio of the magnetic energy and the kinetic energy of the plasma), which takes into account the influence of the magnetic field on the turbulence formation in the flow.

If $\mathrm{Ri}_\Sigma = \mathrm{Pr}_K^{turb}$, here exists the unique real solution $\mathrm{w}_{turb} = 0$ corresponding to the laminar regime. For the turbulent regime and, therefore, $\mathrm{Ri}_\Sigma < \left(\mathrm{Ri}_\Sigma\right)_{cr}$, we obtainfor the turbulent viscosity coefficient of the electro conductive fluid

$$\nu_K^{turb} = \alpha_{ss} L^{*2}\sqrt{2\left(\overset{0}{\boldsymbol{\mathcal{D}}}:\overset{0}{\boldsymbol{\mathcal{D}}}\right)}, \tag{38}$$

where $L^* := L\varphi$; dimensionless function $\varphi := (1 - \mathrm{Ri}_\Sigma / \mathrm{Pr}_K^{turb})^{1/4}$ takes into account the influence of the magnetic field and the inverse effect of heat transport on the turbulence development via the mixing length. Here, the following approximate estimate for the critical Richardson number is obtained: $\left(Ri_\Sigma\right)_{cr} = \mathrm{Pr}_K^{turb} = \bar{\rho}\, c_p \nu_K^{turb} / \lambda^{turb}$.

The known Prandtl–Nikuradze formula can be used for the calculation of the Prandtl mixing length (in the case of an absence of a magnetic field); this formula, as applied to the simulation of the disk structure, can be written in the form

$$L / h_{eff} = 0{,}14 - 0{,}08(1 - z / h_{eff})^2 - 0{,}06(1 - z / h_{eff})^4. \tag{39}$$

The closed system of averaged MHD equations presented in this section is the basis for the simulation of the structure and evolution of the turbulized protoplanetary disk; in the case of the simulation of thin accretion disks, this system can be considerably simplified [Pringle, King, 2007].

4. Modeling the turbulent transport coefficients in an accretion disk

The key problem in astrophysics is related to the mechanism of angular momentum exchange between accretion disc and protostar. The developed thermodynamic model of MHD-turbulence in magnetic field is applicable to numerical modeling of the structure and evolution of accretion discs, in particular those around *T*-Tauri stars giving birth to planets or binary stars with mass transfer (of dwarfs novae systems). Such discs formed by a matter rotating around compact stellar objects and are retained on nearly circular orbit by combined action of gravity and centrifugal forces. Independent of what caused protoplanetary disc to set up, at the early stage of evolution its matter possesses angular momentum sufficient to reside at a circular orbit. At the same time, every disc mass element spirals slowly (accretes) towards mother star, which should result in the angular momentum transfer from the inner regions outward.

Indeed, the existing mass and angular momentum distribution in the Solar system, as well as in other numerous systems of young stars with discs, argue for the necessity to invoke an efficient mechanism of mass and angular momentum redistribution in due course of the planetary system formation. Classical ideas are based on friction mechanism of the angular momentum transfer in the accretion disc from protostar outward and its subsequent distribution in space or, in a case of the binary stars, on mechanism of tidal interaction responsible for angular momentum return back to its original source—satellite star. Note that friction mechanism can be operational only if the disc kinematic viscosity exceeds by many orders of magnitude molecular viscosity to provide the necessary mass and accompanying angular momentum transfer for T –Tauri stars.

Such a great viscosity can be accomplished in a turbulent protoplanetary disc, the turbulent state of disc matter caused by its large-scale shift of velocity in the differential rotation relative gravity center being the key concept in astrophysics [see, e.g., Zel'dovich, 1981; Fridman, 1989; Dubrulle, 1993; Gor'kaviy, Fridman, 1994; Balbus, Hawley, 1991; Richard, Zahn, 1999; Bisnovaty-Kogan, Lovelace, 2001]. Besides, MHD turbulence can contribute significantly to the angular momentum redistribution [Eardley, Lightman, 1975; Galeev et al., 1979; Coroniti, 1981; Tout, Pringle, 1992; Brandenburg et al., 1996; Lesch, 1996], its efficiency as dissipation mechanism depending on magnetic lines reconnection [Kadomtsev, 1987].

It is reasonable to assume that chaotic magnetic fields caused by the turbulent dynamo mechanism or brought by accreting interstellar plasma, occurred in the disc at early stage of its evolution. These fields having energy compared to that of hydrodynamic turbulence, mixed owing to disc matter differential rotation and experienced reconnections at the disc boundary, could contribute substantially to the turbulent viscosity not only in the inner disc regions but also in the outer parts of its corona where the matter is ionized. Let us recall that such a fundamental physical process in space plasma as magnetic lines reconnection responsible for exhibiting many features of plasma activity, is only possible when in an electro-conductive matter magnetic lines of opposite direction come close together. This process results in the high electric current density set up. Note also that before reconnection in plasma having a definite storage of the magnetic energy the so called break (tearing) instability develops which ultimately is responsible for the reconnection and transfer of the magnetic energy into the plasma kinetic or thermal energy.

Alongside with chaotic magnetic fields, large scale well ordered magnetic field in some parts of accretion disc can exist. Due to turbulent transfer such a field extends to at least the disc inner edge. This field penetrates to both sides of the disc vicinity. These outer regions experience magnetic strengths influence for which both small scale disturbances connected with turbulence and large scale shear flows are responsible. This results in not only turbulent viscosity and turbulent diffusion parameters variations but also affects connection with electrodynamics of mean fields. In particular, since in the rotating electro-conductive medium effective magnetic diffusion is accompanied by the turbulent electromotive $\alpha\mathbf{B}$ force set up (the so-called a-effect related ultimately to spirality influence on the magnetic field induction generation), one may expect significant action of turbulent dynamo on the accretion disc structure and evolution. It is known that small scale girotropic (reflection non-invariant turbulence) in the fast rotating disc creates “loops” (α-effect) when any magnetic field force tube acquires the form of a distorted O letter (see Parker, 1955). This magnetic loop is accompanied by the current having either parallel or anti-parallel component relative to the applied mean magnetic field for left or right screw accidental spiral motions, respectively. Energy of Joule heat release due to such currents is a powerful source of heating responsible for the disc corona formation that is comparable by its thickness to the disk itself [Galeev et al., 1979]. This is just the size of “primary loops” produced by the buoyancy force although one should keep in mind that in reality the corona can be even thicker. The reason is that in the process of small loops reconnection, loops of progressively larger size can be formed. Basically, this is the well known inverse cascade in the three-dimensional MHD motions when turbulence possesses both kinematic and magnetic spirality. At the same time, large scale force lines running through the corona and closed in the disc, maintain magnetic connection between distant disc regions. Such a connection may be also responsible for an additional strengths source and hence the disc heating.

We therefore see that because of viscous stresses due to differential disc rotation and turbulent dynamo in plasma the disk corona is heated similar to what occurs in the solar corona. In turn, hot corona gives rise to outflow (streams) of matter and field. Actually, such a stream represents magnetized rotating wind flowing from the accretion disc. Together with matter and magnetic field the wind transfers to infinity significant part of the disc angular momentum allowing the disk to contract slowly. In other words, rotating wind outflow, jointly with viscous friction mechanism, provides another opportunity to transport an excess angular momentum from the inner part of protostar-disc system outward. Note that an effective focusing of the disc matter flow (jets) can be formed by magnetic stresses in the wind.

In order to reconstruct structure and evolution of the protoplanetary accretion disk that surrounded the proto-Sun the representative model involving numerous processes in a conducting disk and its corona is required [Kolesnichenko, Marov, 2006, 2007]. As in the case of a non-conducting disk, modeling the turbulent viscosity coefficient ν_K^{turb} in a plasma disk medium under the action of a magnetic field is one of the problems of paramount importance. This section is devoted to studying this problem.

Averaged magneto-hydrodynamic equations for a turbulent accretion disk . Let us consider the slowly evolving turbulent accretion disk in a vacuum which at time instant t rotates with angular velocity $\Omega(\varpi, z)$ about the $0z$ axis. It will be assumed that the disk is electroconductive and there exists initial large-scale slowly varying axially symmetric magnetic field $\mathbf{B}_{\odot}$ of the proto-Sun whose dipole momentum coincides with the disk rotation axis. Below, cylindrical coordinate system (ϖ, φ, z) will be used and it will be assumed that the central plane of the accretion disk coincides with the equatorial plane of the Sun determined by the condition $z = 0$. Here, we consider the model of the thin axially symmetric $\partial(...)/\partial\varphi = 0$ accretion disk for which the spatial scale of the variation of the structural parameters in the layer perpendicular to the equatorial plane is large, as compared to the disk half thickness, i.e., $h(\varpi)$ is small as compared to ϖ for all ϖ, $\partial h/\partial\varpi \approx h/\varpi << 1$. It can be shown that the accretion disk thickness depends on the balance of heating and cooling (see, e.g., [Shapiro, Teukolsky 1985]). Efficient cooling results in a geometrically thin disk. For such a disk, the character of the flow of the conducting disk matter can be analyzed using 2D MHD equations.

Let us first analyze viscosity law (38) in the thin accretion disk in which the motion of matter can be represented as the superposition of the general differential rotation and the random turbulent motion. It will be assumed for simplicity that the disk rotation is so slow that the meridional circulation can be neglected (see, e.g., [Tassoul 1982]); i.e., the average motion of space matter is realized only in the azimuthal direction, and the true flow velocity pulsates randomly about this average value and changes irregularly in the meridional and azimuthal directions; then, $\langle \mathrm{u}\rangle_\varpi = 0$, $\langle \mathrm{u}\rangle_\varphi = \varpi\Omega(\varpi,z)$, $\langle \mathrm{u}\rangle_z = 0$.

Under these assumptions, the $\varpi\varphi$ component of kinetic Reynolds tensor (24) and dissipative function Φ_{v} take the form

$$R^K_{\varpi\varphi} = \bar{\rho}\,\nu_K^{turb}\varpi\frac{\partial\Omega(\varpi,z)}{\partial\varpi},$$

$$\Phi_{\mathrm{v}} := \nu_K^{turb}2\left(\overset{0}{\mathcal{D}}:\overset{0}{\mathcal{D}}\right) = \nu_K^{turb}\varpi^2\left\{\left(\frac{\partial\Omega(\varpi,z)}{\partial\varpi}\right)^2 + \left(\frac{\partial\Omega(\varpi,z)}{\partial z}\right)^2\right\}. \qquad (40)$$

Then, for the larger part of the disk (except for regions close to the proto-Sun) the following approximate expression for the turbulent viscosity coefficient is valid:

$$\nu_K^{turb} = \alpha_{ss}L^{*2}\varpi\left|\frac{\partial\Omega(\varpi,z)}{\partial\varpi}\right|, \quad L^*(z) := L(z)\left\{1 - \frac{\mathrm{Ri}_K - \mathrm{Ri}_M}{\mathrm{Pr}_K^{turb}}\right\}^{1/4}, \qquad (41)$$

where

$$\mathrm{Ri}_K \cong \frac{\Omega^2_{K,mid}\,z}{\varpi^2}\frac{1}{\langle T\rangle}\frac{\partial\langle T\rangle/\partial z + G_a}{\left(\partial\Omega/\partial z\right)^2}, \quad \mathrm{Ri}_M \cong \frac{1}{4\pi\mu_0\bar{\rho}}\frac{(\partial\bar{B}_\varphi/\partial z)^2}{\varpi^2\left(\partial\Omega/\partial z\right)^2}, \qquad (42)$$

$$G_a \equiv \frac{g_z}{c_p} = -\frac{1}{c_p}\frac{G\mathcal{M}_\odot z}{\varpi^3}\left(1+\frac{z^2}{\varpi^2}\right)^{-3/2} \cong \frac{1-\gamma}{\gamma}\frac{1}{\Re}\Omega^2_{K,mid}\,z \qquad (43)$$

is the adiabatic temperature gradient in the protoplanetary accretion disk. In expressions (42) and (43), the following effective force of gravity is used $\mathbf{g} = \{0, 0, -g_z\}$, where

$$g_z = \frac{G\mathcal{M}_\odot z}{\varpi^3}\left(1+\frac{z^2}{\varpi^2}\right)^{-3/2} \cong \Omega^2_{K,mid}(\varpi)\,z; \quad \Omega_K(\varpi,z) := \sqrt{\frac{G\mathcal{M}_\odot}{(\varpi^2+z^2)^{3/2}}} \qquad (44)$$

is the Keplerian angular velocity and $\Omega_{K,mid}(\varpi) \equiv \Omega_K(\varpi,0) = \sqrt{G\mathcal{M}_{\odot}/\varpi^3}$ is the Keplerian angular rotation velocity in the central plane of the disk.

It can be seen from formula (42) that in the case of adiabatic temperature distribution along the height (indifferent stratification) when $-\partial\langle T\rangle/\partial z = -\left(\partial\langle T\rangle/\partial z\right)_{ad} = = z\Omega^2_{K,mid}(\varpi)/c_P$, the Richardson number $\mathrm{Ri}_K = 0$, i.e., the temperature gradient in the disk does not influence the turbulent transport coefficients. However, in the case of the unstable thermal stratification of the accretion disk when super-adiabatic temperature gradients take place, the turbulent energy increases due to the instability energy in the direction perpendicular to the equatorial plane of the disk (convective source of turbulence); in this case, the turbulent viscosity coefficient simultaneously increases. At the same time, the spatial inhomogeneity (over height) of the averaged magnetic field results in an increase in the turbulent energy, since the magnetic Richardson number $\mathrm{Ri}_M > 0$. The inverse Prandtl–Schmidt number $1/\mathrm{Pr}^{turb}_K$ in formula (41) can be taken as equal to unity if the main turbulence mechanism is shear stress during differential disk rotation; however, it can be larger by a factor of 2–3 if the reason for the turbulence is thermal convection in the vertical direction.

For obtaining the formal matching of expression (41) and the widely used in astrophysical literature Shakura–Sunyaev formula [Shakura, 1972] for simulation of the thin layer situated in the central plane of the Keplerian disk, the following should be assumed in (41): $\mathrm{Ri} = 0$, $\mathrm{Ri}_M = 0$, and the angular velocity of Keplerian rotation $\Omega_{K,mid}(\varpi)$ should be substituted in it. If the effective disk half-thickness $h_{eff} \cong c_s\big|_{z=0}/\Omega_{K,mid}$ (which can be estimated using the force balance in the *z* direction, see below) is taken as the turbulence scale, we obtain

$$\nu^{turb}_K = {}^3\!/_2\,\alpha_{ss} h^2_{eff}\Omega_{K,mid} = {}^3\!/_2\,\alpha_{ss} h_{eff} c_s\big|_{z=0} = {}^3\!/_2\,\alpha_{ss}\gamma\left(\bar{p}/\bar{\rho}\right)\big|_{z=0}/\Omega_{K,mid} \qquad (45)$$

Then, the following dependence holds between the ϖ,φ component of Reynolds turbulent stress tensor $R_{\varpi\varphi}$ and gas pressure $\bar{p}$:

$$R_{\varpi\phi} = \bar{\rho}\nu^{turb}_K \varpi(\partial\Omega_{K,mid}/\partial\varpi) = -\alpha_s\bar{p}\big|_{z=0} \qquad (46)$$

Here, $c_s\big|_{z=0} \cong \sqrt{\gamma\bar{p}/\bar{\rho}}\big|_{z=0}$ is the thermal speed of sound and $\alpha_s = {}^9\!/_4\,\alpha_{ss}$ is the free parameter which cannot be determined more or less precisely and satisfies the following constraint: $\alpha_s \le 1$.

Astrophysical models constructed using relation (46) are related to the so-called viscous α disks. In such models, α_s is usually a free parameter in disk structure equations. The determination of this parameter based on different assumptions on the nature of physical processes in the disk was the topic of multiple studies. The value of Shakura–Sunyaev disk parameter αs characterizing the degree of excitation of turbulent motion can be empirically calibrated, in particular, using timedependent spectra obtained during the observation of flashes with mass transfer in binary stellar systems with dwarf novae. For this case, the values of α_{ss} in the interval $0.1 \le \alpha_s \le 1$ [Lynden-Bell, Pringle, 1974; Bath, Pringle, 1981] were found. These values agree with estimates in [Eardley et al., 1975; Eardley et al., 1978; Heyvaerts et al., 1996; Fridman, Bisikalo, 2008], where the viscosity due to velocity shear and the reconnection of force lines of the chaotic magnetic field was considered. However, for this case the following values were obtained: $0.01 \le \alpha_s \le 1$. In analytical papers [Coroniti, 1981; Tout, Pringle, 1992], the connection between the viscosity in the disk and the process of the reconnection of magnetic fields inside the disk was found.

It is known that the reconnection rate can be characterized by the quantity $\mathrm{M}_A = \mathrm{u}/c_A$, where u is the matter velocity before the discontinuity and $c_A = |\mathbf{B}|/\sqrt{4\pi\mu\rho}$ is the Alfven velocity before the discontinuity [Priest, Forbes, 2005]. Both models use shear flow inside the disk for magnetic field amplification and use MHD turbulence as the mechanism of radial matter transport. In Coroniti's model, Keplerian motion in the disk with time creates a magnetic field in the plane of the disk, forming elliptical cells. These magnetic cells are continuously created and destroyed in the turbulent process, which results in the radial diffusion of the plasma in the disk. The Shakura–Sunyaev viscous parameter obtained in this paper is expressed in terms of the reconnection parameter as $\alpha_{ss} \approx \mathrm{M}_\mathrm{A}^{2/3}$. In the mentioned Tout–Pringle model, no special magnetic field geometry is assumed, but sources and sinks are estimated for different components. It was shown in this model that there exists a reconnection of the vertical field initiated by strong radial shear flows, which is a more important process than the reconnection of the azimuthal field, as it is assumed in Coroniti's model. The Tout–Pringle model yields the following expression for the Shakura–Sunyaev parameter: $\alpha_{ss} \approx 0.6\mathrm{M}_A$. In order to cause accretion which agrees with observations of different astrophysical phenomena, the Mach number M_A should be of an order of 0.1, which assumes very high reconnection rates. However, at present there are no grounds to assume that this fast reconnection is possible in the regime of turbulent MHD motion taking place in the disk. In order to obtain a realistic picture of the relationship between accretion and reconnection, probably, numerical simulation is required; this simulation should consider the turbulent dynamo and reconnection processes in a self-consistent way. Other analytical models providing the calculation of αss are known in published data; however, all of these models cannot be considered proven, since in all of them the result of the simulation of the Shakura–Sunyaev parameter, in essence, is reduced to expressing unknown quantity α_{ss}αss in terms of some other poorly defined quantity.

Note once more that in the Shakura–Sunyaev approach developed specially for thin accretion disks, the inverse influence of the convective heat transport and the gradient of the large-scale magnetic field on the turbulence development was not taken into account. In relation with the adequate simulation of the structure and the evolution of the solar protoplanetary disk and its corona, it seemed reasonable to reject the α formalism and obtain the generalization of formula (46) to the case of the density-layered matter of the disk with finite thickness.

Conclusions

Within the framework of the main problem of cosmogony associated with the reconstruction of the protoplanetary disk of the Sun at the earliest stages of its existence, a closed system of MHD equations of the mean motion scale is formulated, which is intended for the numerical solution of problems of mutually consistent modeling of the structure and evolution of the accretionary protoplanetary disk and the associated corona. A model of a plasma disk of finite thickness is discussed, which takes into account turbulence dissipation due to kinematic and magnetic viscosity and thermal conduction processes. In contrast to the already classical approach of Shakura and Sunyaev, which was developed specifically for thin accretion disks and which did not take into account the back effect of convective heat transfer, as well as the effect of the large-scale magnetic field gradient on disk turbulence, in this paper we propose a new approach to modeling coefficient of turbulent kinematic viscosity for an astrophysical disk, This approach takes into account the influence of the external and generated magnetic fields, as well as the processes of convective heat transfer on turbulence in a stratified layer of finite thickness, and thereby modifies the Shakura–Sunyaev alpha formalism, which is widely used in the astrophysical literature.

This approach opens wide prospects for the further improvement of the mathematical models of the origin and evolution of the Solar System, which is of primary importance for the solution of the problem of the origin of the terrestrial biosphere.

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