ISSN 2071-2898 (Print)
ISSN 2071-2901 (Online)

Ovchinnikov M. Y., Pen'kov V.I., Roldugin D.S.

Spin-stabilized satellite with three-stage active magnetic attitude control system

Recommended form of bibliographic references

Ovchinnikov M. Y., Pen'kov V.I., Roldugin D.S. Spin-stabilized satellite with three-stage active magnetic attitude control system // Keldysh Institute Preprints. 2011. No. 6. 23 p.
URL: http://library.keldysh.ru/preprint.asp?id=2011-6&lg=e

Publications based on the preprint

M.Yu. Ovchinnikov, D.S. Roldugin, V.I. Penkov, Asymptotic study of a complete magnetic attitude control cycle providing a single-axis orientation // Acta Astronautica, 2012, V. 77, pp. 48-60
DOI: 10.1016/j.actaastro.2012.03.001
URL: http://www.sciencedirect.com/science/article/pii/S0094576512000720

KELDYSH INSTITUTE OF APPLIED MATHEMATICS
RUSSIAN ACADEMY OF SCIENCE

M.Yu. Ovchinnikov, V.I. Pen'kov, D.S. Roldugin

Spin-stabilized satellite with three-stage active magnetic attitude control system

Moscow
2011

Spin-stabilized satellite with three-stage active magnetic attitude control system. M.Yu. Ovchinnikov, V.I. Pen’kov, and D.S.Roldugin. The Keldysh Institute of Applied Mathematics of Russian Academy of Sciences, 2011, 23 p., 14 items of bibliography, 9 figures

The angular motion of an axisymmetrical satellite equipped with the active magnetic attitude control system is considered. Dynamics of the satellite is analytically studied on the whole control loop consisting of a bundle of three successive algorithms. Those algorithms are as follows: nutation damping, spinning about the axis of symmetry, reorientation of the axis in the inertial space. In certain cases explicit solutions of the equations of motion are obtained. The results are verified by numerical simulation.

Key words: active magnetic attitude control, spin-stabilized satellite, averaged geomagnetic field model, time-response

Спутник с активной магнитной системой ориентации, стабилизируемый собственным вращением в три этапа. М.Ю.Овчинников, В.И. Пеньков, Д.С.Ролдугин. ИПМ им.М.В.Келдыша РАН, Москва, 2011г., 23 с., библиография: 14 наименований, 9 рисунков

Рассматривается осесимметричный спутник-гироскоп, оснащенный активной магнитной системой ориентации, реализующей последовательно три закона управления, которые позволяют установить ось симметрии спутника в заданном направлении в инерциальном пространстве. Исследуются три алгоритма: гашение нутационных колебаний, раскрутка вокруг оси симметрии и переориентация оси симметрии в инерциальном пространстве. В рамках осредненной модели геомагнитного поля проводится аналитическое исследование уравнений движения спутника для всех трех законов управления. Анализируется зависимость их быстродействия от параметров задачи.

Ключевые слова: магнитная система ориентации, спутник, стабилизируемый собственным вращением, осредненная модель магнитного поля Земли, быстродействие системы ориентации

Introduction

Spin stabilization is a common way to maintain a satellite attitude. Satellite acquires the property of a gyroscope while it is spinned around the axis of symmetry with a high angular velocity. Only spinning around the principal axis of maximum inertia is stable if the satellite is equipped with an energy dissipation device and it is no torque subjected [1]. This result is important since an energy dissipation device is necessary for any attitude control system including one for a spinning satellite. In the latter case attitude control system performance may be divided into three modes: angular velocity damping, spinning around the axis of symmetry, reorientation of the spin-axis to a required direction in the inertial space. These modes may be implemented consequently or combined. To conduct the whole control circle, satellite must be equipped with an active attitude control system to manage its angular velocity and attitude.

In this paper we consider the most common way of attitude control of a spinning satellite. The method is based on the interaction between the geomagnetic field and satellite magnetized actuators. Magnetic attitude control systems (MACS) are especially used when it is critical to have low-cost and low-mass control system capable of implementing conventional algorithms for onboard computer. Principal methods of magnetic attitude control of a spinning satellite are considered in [2] and [3], in [4] general dynamical properties of a spin-stabilized satellite along with technical issues are discussed. Paper [5] is a comprehensive survey of works on satellite orientation and stabilization, steady-state motion stability and external torques effect including these problems for a spinning satellite.

Five different algorithms are studied in this paper. “*-Bdot*” algorithm implemented by three coils is used for the initial angular velocity damping and by one coil is used as a damper of nutational motion. The algorithm is used simultaneously with the coarse reorientation algorithm or fine reorientation algorithm. Then spinning-up around the axis of symmetry is implemented (if still necessary) and fine reorientation is carried out. Dynamics of the satellite is studied using averaging technique [6] for each algorithm and motion state. Results obtained allow us to analyze the dependence of primary satellite motion characteristics on the orbit inclination and other parameters of the satellite. In specific cases we could obtain preferable values of the parameters from the time-response point of view.

1. Problem description

Summarize all assumptions, introduce geomagnetic field model, reference frames, equation of motion and analysis technique.

Angular motion of a spinning satellite equipped with MACS is examined in the paper. MACS contains three mutually orthogonal magnetic coils. Assume that MACS is capable to develop a magnetic dipole moment in arbitrary direction but of limited value wrt satellite. Only torque produced by the interaction of MACS with the geomagnetic field is taken into account. Among number of geomagnetic field models available we chose the averaged one to represent the geomagnetic field [7]. Angular motion of a satellite is described by the Beletsky-Chernousko variables [8]. Satellite's orbit is assumed as a Keplerian circular one. MACS implements following algorithms:

1. Nutation damping. Only single coil disposed along the axis of symmetry of a satellite is used. It implements cut "*-Bdot*" algorithm.
2. Spinning of the satellite around its axis of symmetry. Two coils disposed in its equatorial plane (i.e. plane perpendicular to the axis of symmetry) are used.
3. Fine spin axis reorientation in the inertial space.

The choice of geomagnetic field model is one of the most crucial points for the success of whole work. Let us describe geomagnetic field model used in this paper. Magnetic induction vector is usually determined using decomposition to the Gauss series [9]

$$\mathbf{B} = \nabla V,\ V = -R\sum_{i=1}^{k}\left(\frac{R}{r}\right)^{i+1}\sum_{n=0}^{m}\left(g_n^m(t)\cos m\lambda_0 + h_n^m(t)\sin m\lambda_0\right)P_n^m(\cos\vartheta)$$

where λ_0 is the longitude of the point where the induction vector is calculated, $\vartheta = 90^\circ - \theta_0$, θ_0 is the latitude of the point, r is the distance to the point from the Earth center, R is average Earth radius. g_n^m and h_n^m are Schmitt coefficients given in a table, P_n^m is a quasinormalised Legendre polynomial. It is impossible to use this model in the analytical study. So, a number of consequent simplifications are introduced. *Inclined dipole model* is derived from the Gauss model when one takes into account only three first terms. It describes the field of a dipole placed in the Earth center and inclined to its axis by 168°26'. This model admits rather compact analytical expression though it is still too complicated for the analytical study. Further simplification, the *direct dipole model* is wide used. In this model the geomagnetic field is one of the dipole placed in the center of the Earth and directed antiparallel to its axis of day rotation. Magnetic induction vector moves almost uniformly on the near-circular cone side while a satellite moves along the orbit. This model, though rather simple and suitable and allow to utilize powerful *Floquet theory* to study periodical solutions appeared from motion equations, nevertheless, does not allow us to get the solution of the equations of motions in terms of explicit formulas or quadratures. So, we go further on a way of simplification and compromising between complexity and veracity and introduce one more simplification considering the geomagnetic induction vector as moving uniformly on the circular cone side and its magnitude is constant. To do this we need to notify a reference system $O_aY_1Y_2Y_3$ where O_a is the Earth center, O_aY_3 axis is directed along with the Earth axis, O_aY_1 lies in the Earth equatorial plane and is directed to the ascending node of the satellite orbit, O_aY_2 axis is directed so the whole system to be a right-handed. If the magnetic induction vector source point is translated to the O_a then the cone is tangent to the O_aY_3 axis, its axis lies in the $O_aY_2Y_3$ plane (Fig. 1). The cone half-opening angle is given [7] by

$$\mathrm{tg}\,\Theta = \frac{3\sin 2i}{2\left(1 - 3\sin^2 i + \sqrt{1 + 3\sin^2 i}\right)} \qquad (1.1)$$

where i is the orbit inclination. The geomagnetic induction vector moves uniformly on the cone side with the doubled orbital angular speed, $\chi = 2u + \chi_0$ where is the argument of latitude, ω_0 is the orbital angular velocity. Without loss of generality we can assume $\chi_0 = 0$.

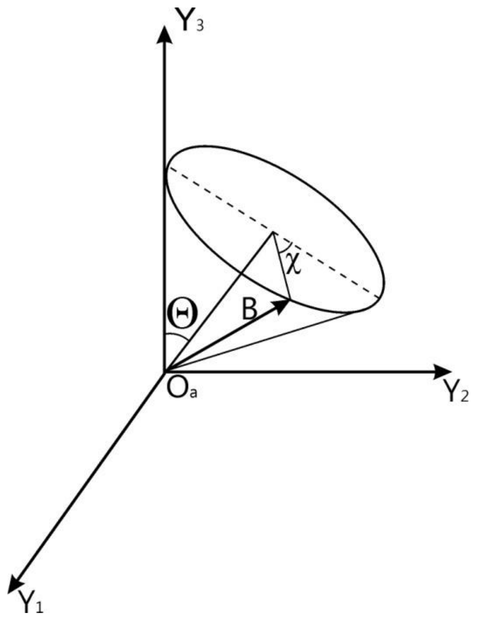

Fig. 1. Averaged geomagnetic field model

This model, sometimes called *averaged*, is used in our work. It does not allow us to take into account non-uniformity of geomagnetic induction vector motion (as right dipole model does) and its diurnal change (as inclined dipole model does) but it is considered as a good trade-off between the accuracy of modeling geomagnetic field and the possibility to get analytical result.

Angle Θ is of great importance for our work. Expression (1.1) introduces the relationship between Θ and the orbit inclination. In fact, these angles are close, so we may consider $\Theta \approx i$ for a qualitative analysis of the system time-response with respect to the orbit inclination since the maximum value of $\Theta - i$ is about 10°. Comprehensive comparison of models can be found in [9].

Let us introduce all necessary reference frames.

$O_aZ_1Z_2Z_3$ is the inertial frame, got from $O_aY_1Y_2Y_3$ turning by angle Θ about O_aY_1 axis.

$OL_1L_2L_3$ is the frame associated with the angular momentum of a satellite. O is the satellite center of mass, OL_3 axis is directed along the angular momentum, OL_2 axis is perpendicular to OL_3 and lies in a plane parallel to the $O_aZ_1Z_2$ plane and containing O, OL_1 is directed such that the reference frame is right-handed.

$Ox_1x_2x_3$ is the bound frame, its axes are directed along the principal axes of inertia of the satellite.

Reference frames mutual orientation is described with the direct cosine matrices $\mathbf{Q}, \mathbf{A}$ expressed in the following tables

$$
\begin{array}{cccc}
 & L_1 & L_2 & L_3 \\
Z_1 & q_{11} & q_{12} & q_{13} \\
Z_2 & q_{21} & q_{22} & q_{23} \\
Z_3 & q_{31} & q_{32} & q_{33}
\end{array},
\qquad
\begin{array}{cccc}
 & x_1 & x_2 & x_3 \\
L_1 & a_{11} & a_{12} & a_{13} \\
L_2 & a_{21} & a_{22} & a_{23} \\
L_3 & a_{31} & a_{32} & a_{33}
\end{array}.
$$

We introduce low indices Z, L, x to denote the vector components in frames $O_aZ_1Z_2Z_3$, $OL_1L_2L_3$ and $Ox_1x_2x_3$ respectively. For example, for the first component of a torque in these frames we write M_{1Z}, M_{1L}, M_{1x}.

We use the Beletsky-Chernousko variables to represent the motion of the satellite. These variables are $L, \rho, \sigma, \varphi, \psi, \theta$ [8] where L is the angular momentum magnitude, angles ρ, σ represent its orientation with respect to $O_aZ_1Z_2Z_3$ frame (Fig. 2). Orientation of the frame $Ox_1x_2x_3$ with respect to $OL_1L_2L_3$ is described using Euler angles φ, ψ, θ. The same variables were first introduced by Bulgakov [11] for a gyroscope movement problem. The equations for an axisymmetrical satellite were first proposed by Beletsky [12] and for a satellite with arbitrary moments of inertia by Chernousko [13]. Equations of a free rigid body motion about its center of mass in variables φ, ψ, θ were first proposed by Wittaker [13] but evolutionary equations were not considered. Direct cosine matrix $\mathbf{Q}$ takes form

$$
\mathbf{Q} = \begin{pmatrix} \cos\rho\cos\sigma & -\sin\sigma & \sin\rho\cos\sigma \\ \cos\rho\sin\sigma & \cos\sigma & \sin\rho\sin\sigma \\ -\sin\rho & 0 & \cos\rho \end{pmatrix}. \qquad (1.2)
$$

Direct cosine matrix $\mathbf{A}$ is a common transition matrix for Euler angles and is of the form

$$\mathbf{A}=\begin{pmatrix}\cos\varphi\cos\psi-\cos\theta\sin\varphi\sin\psi & -\sin\varphi\cos\psi-\cos\theta\cos\varphi\sin\psi & \sin\theta\sin\psi\\ \cos\varphi\sin\psi+\cos\theta\sin\varphi\cos\psi & -\sin\varphi\sin\psi+\cos\theta\cos\varphi\cos\psi & -\sin\theta\cos\psi\\ \sin\theta\sin\varphi & \sin\theta\cos\varphi & \cos\theta\end{pmatrix}.(1.3)$$

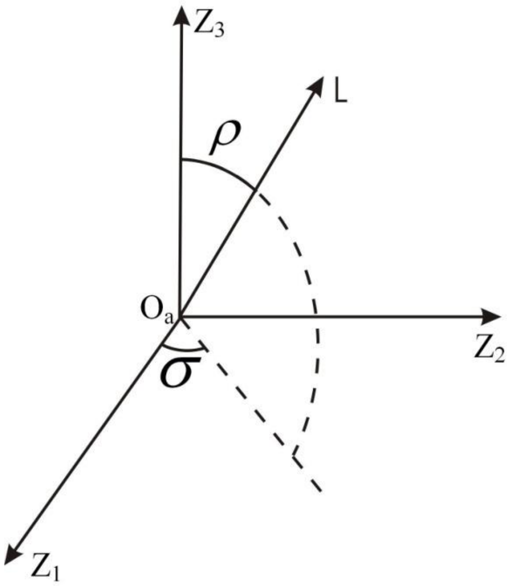

Fig. 2. Angular momentum attitude in the inertial space

Inertia tensor of the satellite is $\mathbf{J}_x = diag(A, A, C)$. Angular motion of the satellite in a circular Keplerian orbit is described [8] by the equations

$$\frac{dL}{dt}=M_{3L},\ \frac{d\rho}{dt}=\frac{1}{L}M_{1L},\ \frac{d\sigma}{dt}=\frac{1}{L\sin\rho}M_{2L},$$

$$\frac{d\theta}{dt}=\frac{1}{L}\left(M_{2L}\cos\psi-M_{1L}\sin\psi\right),$$

$$\frac{d\varphi}{dt}=L\cos\theta\left(\frac{1}{C}-\frac{1}{A}\right)+\frac{1}{L\sin\theta}\left(M_{1L}\cos\psi+M_{2L}\sin\psi\right), \qquad (1.4)$$

$$\frac{d\psi}{dt}=\frac{L}{A}-\frac{1}{L}M_{1L}\cos\psi\,\mathrm{ctg}\,\theta-\frac{1}{L}M_{2L}\left(\mathrm{ctg}\,\rho+\sin\psi\,\mathrm{ctg}\,\theta\right)$$

where M_{1L}, M_{2L}, M_{3L} are the torque components in $OL_1L_2L_3$ frame.

Equations of motion in Beletsky-Chernousko variables are convenient for the asymptotical methods implementation. If the torque acting on the satellite is small in a sense of a small ratio between angular momentum change during one orbit or one revolution about its center of mass and its mean value on this interval then small parameter ε may be introduced. Equations (1.4) are of the form

$$\frac{d\mathbf{x}}{dt} = \varepsilon \mathbf{X}(\mathbf{x}, \mathbf{y}, t), \frac{d\mathbf{y}}{dt} = \mathbf{y}_0(\mathbf{x}) + \varepsilon \mathbf{Y}(\mathbf{x}, \mathbf{y}, t) \qquad (1.5)$$

where $\mathbf{y} = (\varphi, \psi, u)$ are fast variables, while $\mathbf{x} = (l, \rho, \sigma, \theta)$ are slow ones. So, we can use averaging technique [6] to determine slow variables evolution. In order to do it we need to average equations in the vicinity of the undisturbed solution of equations (1.4). However, since this motion is a regular precession, we need only to average separately the equations for slow variables over fast variables. After this we get evolutionary equations for slow variables with accuracy of the order of ε on the time interval of order of $1/\varepsilon$. It also should be noted that it is necessary only to average over φ and u since the satellite is considered axisymmetrical.

2. Nutation damping algorithm

Nutation damping algorithm is constructed on the basis of well-known "*-Bdot*" damping control. If the polar component of the angular velocity is less than or equal to the necessary value, it is impractical to damp it and then spin again. In this situation only equatorial component should be damped. In order to do so, we use the "*-Bdot*" algorithm implemented by a single coil only. Magnetic dipole moment of the satellite $\mathbf{m}_x = (0, 0, m)^T$ in this case is determined by the expression

$$\mathbf{m}_x = -k_1 \left(\frac{d\mathbf{B}_x}{dt} \mathbf{e}_3 \right) \mathbf{e}_3, \qquad (2.1)$$

where k_2 is a new positive coefficient, $\mathbf{e}_3$ is a unit vector of the axis of symmetry of the satellite. The geomagnetic induction vector derivative in $Ox_1x_2x_3$ frame may be obtained from its derivative in $O_aZ_1Z_2Z_3$ frame according to the relation

$$\frac{d\mathbf{B}_x}{dt} = \mathbf{A}^T \frac{d\mathbf{B}_Z}{dt} - \boldsymbol{\omega}_x \times \mathbf{B}_x. \qquad (2.2)$$

Nutation damping algorithm is used for the fast rotating satellite in our case. That means we can neglect the first term on the right side of (2.2). It describes geomagnetic induction vector change in the inertial space where it rotates with the velocity of the order of the orbital one. For a fast rotating satellite ($L/A >> \omega_0$, $L/C >> \omega_0$) the torque takes form

$$\mathbf{m}_x = k_1\left(\left(\boldsymbol{\omega}_x \times \mathbf{B}_x\right)\mathbf{e}_3\right)\mathbf{e}_3.$$

In order to obtain dimensionless equations of the form (1.5) from (1.4) we introduce dimensionless torque $\overline{\mathbf{M}}_L$ defined by

$$\mathbf{M}_L = \frac{k_1 B_0^2}{\omega_0 C}\overline{\mathbf{M}}_L. \tag{2.3}$$

Next we introduce argument of latitude $u = \omega_0(t - t_0)$ instead of dimensional time in (1.4) and dimensionless angular momentum l according to the expression $L = L_0 l$ where L_0 is the initial angular momentum magnitude. This leads to (1.4) being rewritten as

$$\begin{gathered}
\frac{dl}{du} = \varepsilon \overline{M}_{3L},\ \frac{d\rho}{du} = \varepsilon\frac{1}{l}\overline{M}_{1L},\ \frac{d\sigma}{du} = \varepsilon\frac{1}{l\sin\rho}\overline{M}_{2L}, \\
\frac{d\theta}{du} = \varepsilon\frac{1}{l}\left(\overline{M}_{2L}\cos\psi - \overline{M}_{1L}\sin\psi\right), \\
\frac{d\varphi}{du} = \eta_1 l\cos\theta + \varepsilon\frac{1}{l\sin\theta}\left(\overline{M}_{1L}\cos\psi + \overline{M}_{2L}\sin\psi\right), \\
\frac{d\psi}{du} = \eta_2 l - \varepsilon\frac{1}{l}\overline{M}_{1L}\cos\psi\,\mathrm{ctg}\,\theta - \varepsilon\frac{1}{l}\overline{M}_{2L}\left(\mathrm{ctg}\,\rho + \sin\psi\,\mathrm{ctg}\,\theta\right).
\end{gathered} \tag{2.4}$$

Here notations $\varepsilon = \dfrac{k_1 B_0^2}{\omega_0 A}$, $\eta_1 = \dfrac{L_0}{\omega_0}\left(\dfrac{1}{C} - \dfrac{1}{A}\right)$, $\eta_2 = \dfrac{L_0}{A\omega_0}$ are introduced, their meaning is explained in section 1. We assume that moments of inertia A and C provide no resonance between η_1, η_2 and 1 (u rate of change) .

Dimensionless equations averaging leads to

$$\frac{dl}{du} = -\frac{1}{2}\varepsilon l\left[2p + (1-3p)\sin^2\rho\right]\sin^2\theta,$$
$$\frac{d\rho}{du} = \frac{1}{2}\varepsilon(3p-1)\sin\rho\cos\rho\sin^2\theta,$$
$$\frac{d\theta}{du} = -\frac{1}{2}\varepsilon\left[2p + (1-3p)\sin^2\rho\right]\sin\theta\cos\theta, \qquad (2.5)$$
$$\frac{d\sigma}{du} = 0$$

In order not to introduce new parameter, small parameter has the same notation but different expression, as it will be for each different algorithm. Equations (2.5) admit substitution $\rho \to -\rho, \rho \to \pi - \rho$ and $\theta \to -\theta, \theta \to \pi - \theta$. It is impossible to obtain the solution of (2.5) in terms of explicit formulas. Let us first consider two special cases representing two stationary solutions for ρ. Trivial equation for σ is omitted from now on.

1. Initial condition $\rho_0 = 0$. Equations (2.5) take form

$$\frac{dl}{du} = -\varepsilon p l \sin^2\theta,$$
$$\frac{d\theta}{du} = -\varepsilon p \sin\theta\cos\theta. \qquad (2.6)$$

Their solution is

$$|\mathrm{tg}\,\theta| = \exp(-\varepsilon p u + c_0),$$

$$l = \frac{1 + \exp(-2p\varepsilon u + 2c_0)}{1 + \exp(2c_0)}$$

where $c_0 = \ln \mathrm{tg}\,\theta_0$. It is clear that with inclination (and, therefore, with p) rise the time-response also increases. Note that modulus in the last expression for θ may be omitted. We will consider $\theta \in [0, \pi/2]$ for further analysis. There is no generality loss since the equations (2.5) admit substitution $\theta \to -\theta, \theta \to \pi - \theta$.

2. Case $\rho_0 = \pi/2$. The equations are similar to (2.6), their solution is

$$\operatorname{tg}\theta = \exp\left[-\varepsilon(1-p)u + c_0\right],$$

$$l = \frac{1+\exp\left[-2(1-p)\varepsilon u + 2c_0\right]}{1+\exp(2c_0)}.$$

Contrary to the previous case, the time response lowers while inclination rises.

Consider now general case. Divide the first equation in (2.5) by the third one and get

$$\frac{dl}{l} = \operatorname{tg}\theta d\theta .$$

That leads to solution $l\cos\theta = \cos\theta_0$. The first integral $I_1(l,\theta) = l\cos\theta$ is obtained. It shows that the third component of angular velocity vector conserves in the fixed frame. Divide now the second equation in (2.5) by the third one and get

$$-\frac{2p+(1-3p)\sin^2\rho}{(3p-1)\sin\rho\cos\rho}d\rho = \operatorname{tg}\theta d\theta$$

which leads to the first integral

$$I_2(\rho,\theta) = \frac{1}{2}(3p-1)\ln\left(\operatorname{tg}^2\rho + 1\right) - 2p\ln\left|\operatorname{tg}\rho\right| + (3p-1)\ln\left|\cos\theta\right|.$$

Now two first integrals satisfying conditions of the implicit function theorem are found and the solution of (2.5) is obtained in quadratures.

There are three parameters which influence the time-response: i, ρ_0, θ_0. As expected, with θ_0 rise the time response (the time necessary to lower θ, and, therefore, the equatorial component of angular velocity $l\sin\theta / A$) falls. Parameter i and ρ_0 effect is presented in Fig. 3 and Fig. 4.

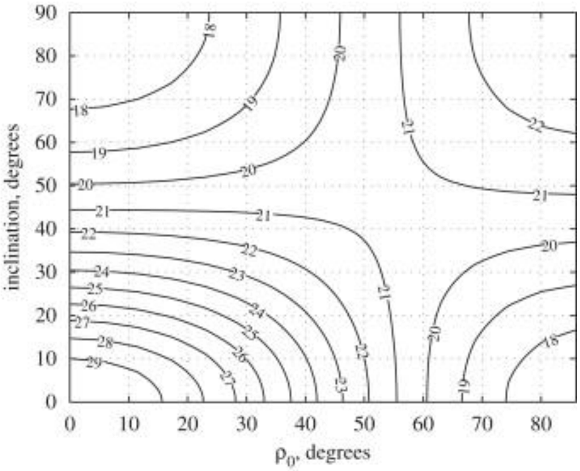

Fig. 3. Angle θ after 2 orbits, θ_0=30°.

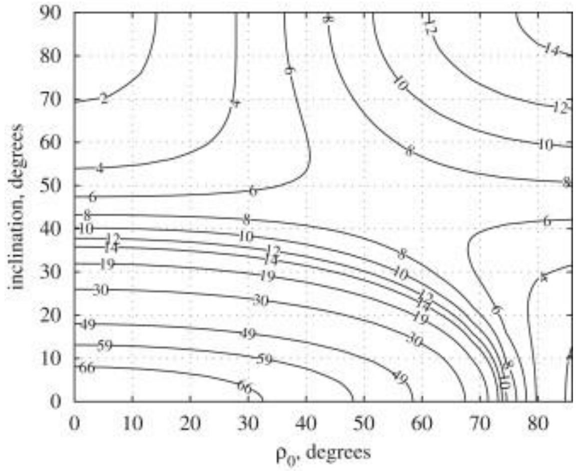

Fig. 4. Angle θ after 15 orbits, θ_0=70°.

As seen from Fig. 3, if ρ_0 is less than approximately 50˚ the time response rises with inclination rise, and if ρ_0 is greater, it falls. Fig. 4 shows that if the algorithm is considered on greater time interval, there is some area where the best inclination (the one that provides minimum θ) is about 45˚. However, it is clear that for the greater inclination θ would not exceed 14˚, while for the small inclination it may stay almost constant. So, it is preferable to use high-inclined orbit.

3. Spinning around the axis of symmetry

In order to obtain the gyro property the satellite is spun around its axis of symmetry. In most cases initial spinning rate after the separation from the launch vehicle is already great, and previous analysis shows that it does not change. However it is valid for the ideal case without disturbing torques, actuators and sensors errors, dynamic disballance in the satellite. The algorithm designed to increase the spinning rate is necessary. Magnetic dipole moment

$$\mathbf{m}_x = k_5 \left(B_{2x}, \quad -B_{1x} \quad 0 \right)^T \tag{3.1}$$

is used. The third component of angular velocity rises since

$$C \frac{d\omega_3}{dt} = k_5 \left(B_{1x}^2 + B_{2x}^2 \right).$$

We use Beletsky-Chernousko variables again. To do that we rewrite (3.1) in a form

$$\mathbf{m}_x = -k_5 \mathbf{e}_3 \times \mathbf{B}_x$$

The torque in a fixed frame is as following

$$\mathbf{M}_x = k_5 \left[B_0^2 \mathbf{e}_3 - \mathbf{B}_x \left(\mathbf{B}_x \mathbf{e}_3 \right) \right].$$

Taking in to account $\mathbf{B}_x = \mathbf{A}^T \mathbf{B}_L$ we get it in $OL_1L_2L_3$ frame,

$$\mathbf{M}_L = k_5 \begin{pmatrix} a_{13}B_0^2 - a_{13}B_{1L}^2 - a_{23}B_{1L}B_{2L} - a_{33}B_{1L}B_{3L} \\ a_{23}B_0^2 - a_{13}B_{1L}B_{2L} - a_{23}B_{2L}^2 - a_{33}B_{2L}B_{3L} \\ a_{33}B_0^2 - a_{13}B_{1L}B_{3L} - a_{23}B_{2L}B_{3L} - a_{33}B_{3L}^2 \end{pmatrix}.$$

Let this torque be small again. Then all the reasoning related to the asymptotical methods holds and averaged equations are

$$\begin{aligned}
\frac{dl}{du} &= \varepsilon \left[2p + (1-3p)\sin^2\rho \right] \cos\theta, \\
\frac{d\rho}{du} &= -\varepsilon \frac{1}{l} (3p-1) \sin\rho \cos\rho \cos\theta, \\
\frac{d\theta}{du} &= -\frac{1}{2l} \varepsilon \left[2 - 2p - (1-3p)\sin^2\rho \right] \sin\theta, \\
\frac{d\sigma}{du} &= 0
\end{aligned} \tag{3.2}$$

where $\varepsilon = \dfrac{k_5 B_0^2}{\omega_0 L_0}$ is a new small parameter. Note that θ is not necessarily small for equations (3.2) to be true. Consider it small after the nutation damping algorithm implementation. From (3.2) we have

$$\frac{dl}{du} = \varepsilon\left[2p + (1-3p)\sin^2\rho\right],$$

$$\frac{d\rho}{du} = -\varepsilon\frac{1}{l}(3p-1)\sin\rho\cos\rho, \qquad (3.3)$$

$$\frac{d\theta}{du} = -\frac{1}{2l}\varepsilon\left[2-2p-(1-3p)\sin^2\rho\right]\theta.$$

If θ is close to π the satellite should be spun in the opposite direction since $\omega_3(0)<0$. This case can be studied in the same way. Equation for θ is separated. From the first and second equations in (3.3) we have the first integral

$$I_1(l,\rho) = (3p-1)\ln l - \frac{1}{2}(3p-1)\ln\left(\mathrm{tg}^2\rho+1\right) + 2p\ln\left|\mathrm{tg}\,\rho\right|.$$

The solution in quadratures is obtained after solving the third equation in (3.3) directly. Note that in case $3p-1=0$ equations lead to $l = 2/3\varepsilon u$. Consider two special cases.

1. If $\rho_0 = 0$ then $\rho = \rho_0$ (stationary solution) and $l = 2\varepsilon pu$. The time-response rises with orbit inclination rise.

2. If $\rho_0 = \pi/2$ then $l = \varepsilon(1-p)u$ and the time response falls with orbit inclination rise.

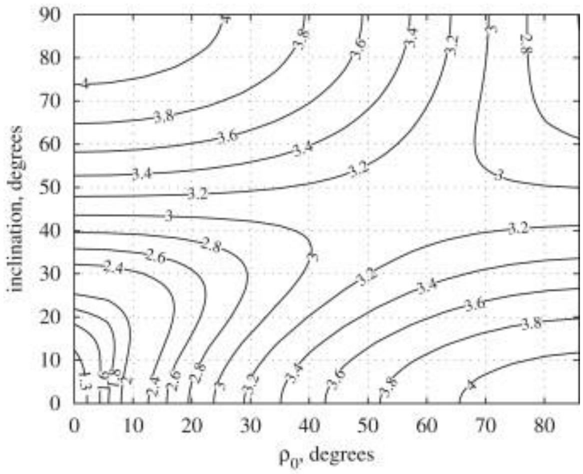

Fig. 5. Angular momentum after 5 orbits, θ_0=1°

Fig. 5 brings the effect of ρ_0 and i on the time-response. For small ρ_0 raising the inclination results in the time-response rise (special case 1), for ρ_0 close to 90˚, the time-response falls with orbit inclination rise (special case 2). However, high-inclined orbit is preferable again since the worst angular momentum magnitude is higher than for low-inclined orbit.

Equatorial component of angular velocity does not rise, its derivative is

$$\frac{d\left(l\sin\theta\right)}{du}=\varepsilon\left[-2+2p+\left(1-3p\right)\sin^2\rho\right]\sin\theta\cos\theta .$$

Since θ is close to 0 equatorial component lowers. Note that the nutation damping algorithm may be implemented simultaneously with the spinning algorithm.

4. Reorientation of the axis of symmetry

Consider the algorithm that brings the satellite rotating fast around its axis of symmetry, to the desired attitude of this axis in the inertial space. The algorithm is

$$\mathbf{m}_x=\left[0,0,k_4\left(\Delta\mathbf{L}\cdot\left[\mathbf{e}_3\times\mathbf{B}\right]\right)\right]^T \qquad (4.1)$$

where $\Delta\mathbf{L}=\mathbf{S}-\mathbf{L}$, $\mathbf{S}$ is the necessary direction of the axis of symmetry, k_4 is a positive constant. Satellite dynamics is described using the Beletsky-Chernousko variables. So, we need to determine the torque in $OL_1L_2L_3$ frame. In this case we have $\mathbf{e}_{3L}=\left(0,0,1\right)^T$ since the satellite is spinned around the axis of symmetry, so $A\left|\omega_i\right|<<C\left|\omega_3\right|$ ($i=1,2$) and its angular momentum is directed almost along this axis

(if it is antiparallel the analysis is similar). Again, the magnetic dipole moment in $Ox_1x_2x_3$ frame has the form $(0,0,m)^{\mathrm{T}}$, and it has the same form in $OL_1L_2L_3$ frame. That leads to $\mathbf{M}_L = B_0\left(-B_{2L}m, B_{1L}m, 0\right)^T$

where $m = k_4\left(\Delta\mathbf{L}\cdot\left[\mathbf{e}_3\times\mathbf{B}\right]\right) = k_4L_0B_0\left(S_{2L}B_{1L} - S_{1L}B_{2L}\right)$.

Equations of motion (1.4) take a form

$$\frac{dl}{du} = 0,$$

$$\begin{aligned}
\frac{d\rho}{du} &= \frac{1}{2}\varepsilon\sin^2\Theta\frac{1}{l}\left(S_1\cos\rho\cos\sigma + S_2\cos\rho\sin\sigma - S_3\sin\rho\right),\\
\frac{d\sigma}{du} &= \frac{1}{2}\varepsilon\sin^2\Theta\frac{1}{l\sin\rho}\left(-S_1\sin\sigma + S_2\cos\sigma\right),
\end{aligned} \tag{4.2}$$

$$\frac{d\theta}{du} = 0$$

where $\varepsilon = \frac{k_4B_0^2}{\omega_0}$ is a small parameter. Equations (4.2) can be solved if $S_1 = S_2 = 0$. That means the axis of symmetry should be oriented along the axis of the cone in the averaged geomagnetic field model. The only non-trivial equation appears

$$\frac{d\rho}{du} = -\eta\sin\rho$$

where $\eta = 0.5\varepsilon S_3\sin^2\Theta$ (note that from the first equation in (4.2) we have $l\equiv 1$). Its solution is

$$\rho = 2\operatorname{arc\,tg}\left[c_0\exp\left(-\eta u\right)\right]. \tag{4.3}$$

Here $c_0 = \operatorname{tg}\rho_0/2$. Fig. 6 introduces ρ for different orbit inclinations in the range from 0 to $\pi/2$. The time-response (time necessary to reorient the axis of symmetry) rises while orbit inclination rises. Angle ρ tends to 0 which corresponds to the necessary satellite attitude.

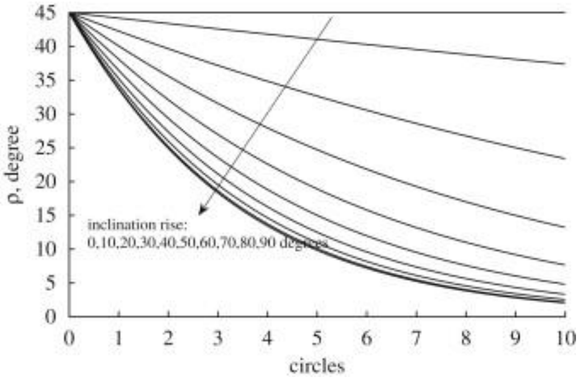

Fig. 6. Angle ρ with respect to time

In case $S_2 = 0$, $S_3 = 0$ (direction to the perigee of the orbit) one can obtain the first integral of (4.2). Divide the second equation in (4.2) by the third one

$$\frac{d\rho}{d\sigma} = \frac{\sin\rho\left(L_1\cos\rho\cos\sigma + L_2\cos\rho\sin\sigma - L_3\sin\rho\right)}{-L_1\sin\sigma + L_2\cos\sigma}.$$

For that particular direction

$$\frac{d\rho}{d\sigma} = -\frac{\sin\rho\cos\rho\cos\sigma}{\sin\sigma},$$

which leads to $I_0 = \mathrm{tg}\,\rho\sin\sigma$ and (4.2) is solved in quadratures.

Form (4.3) we conclude that the high-inclined orbit is definitely preferable.

5. Numerical analysis

Numerical simulation of the satellite motion with algorithms (2.1), (3.1) and (4.1) in fine reorientation mode for successive implementation is shown in Fig. 7, 8 and 9.

Numerical simulation was conducted with gravitational torque taken into account. Sensor measurements are modeled with small errors. Magnetic dipole moment implemented by coils is discrete one, dipole moment $\mathbf{m}$ is substituted by $m_0\,\mathrm{sign}\left(\mathbf{m}\right)$. Following assumptions were made

- Magnetometer and sun sensor are used. Maximum error for sun sensor is 1°, for magnetometer is $4\cdot10^{-7}$ T;
- Inertia tensor $\mathbf{J} = diag\left(0.011, 0.011, 0.02\right)$ kg·m^2;

- Dipole moment used for nutation damping is 0.8 A·m^2, for spinning is 0.1 A·m^2, for reorientation is 0.8 A·m^2, for nutation damping during spinning is 0.2 A·m^2;
- Necessary axis of symmetry attitude in $O_aY_1Y_2Y_3$ frame is $(1,1,0)^T$;
- Initial angular velocity is $(0.1, 0.1, 0.01)^T$ s^{-1};
- Orbit inclination is 60°;
- Right dipole moment is used to represent geomagnetic induction vector.

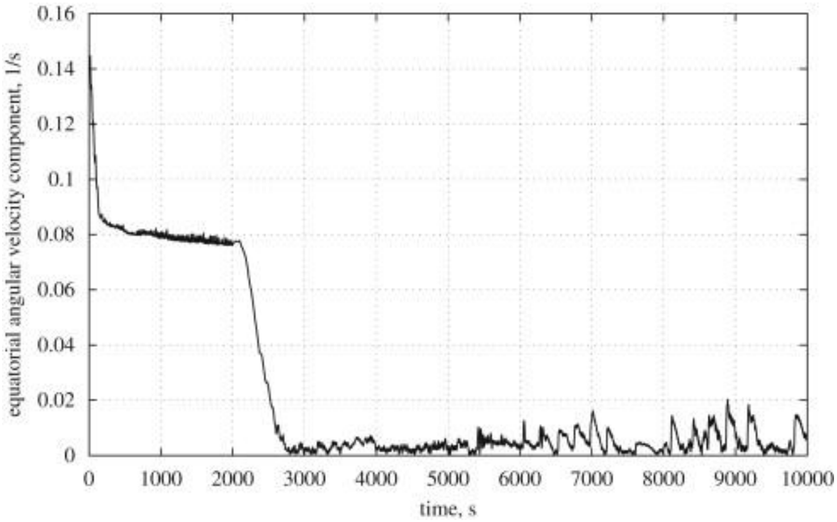

Fig. 7. Nutation damping

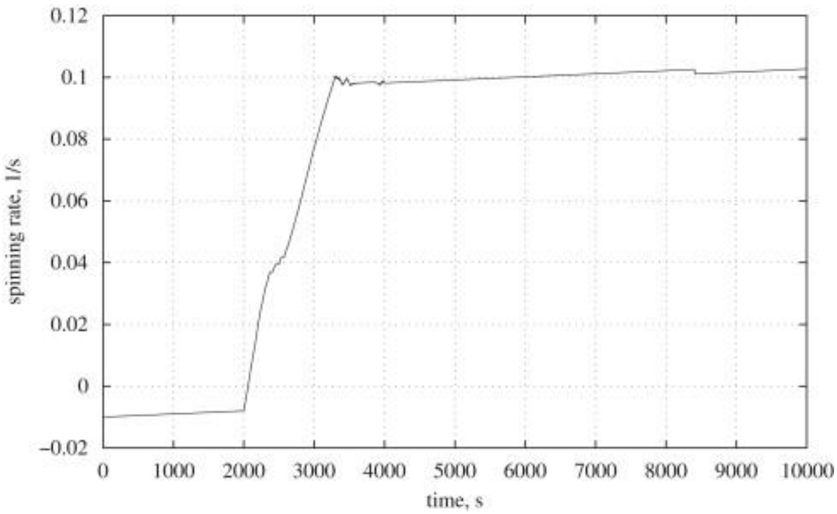

Fig. 8. Spinning around the axis of symmetry

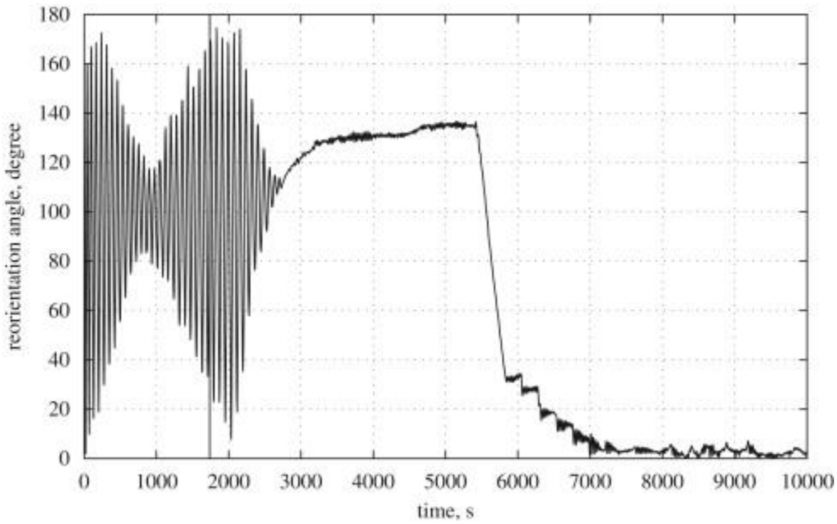

Fig. 9. Fine reorientation in the inertial space

More thorough numerical analysis for the real satellite can found, for example, in [18].

6. Conclusion

The orientation of the axis of symmetry of a spinning satellite in a given direction in the inertial space using proposed set of algorithms for the magnetic attitude control system is shown. Averaged technique is effectively used to obtain evolutionary equations and their first integrals in the frame of averaged geomagnetic field model. This allowed the effect of the orbit inclination and initial conditions on the satellite dynamics to be studied.

Damping of the equatorial component or all components of the angular velocity is proved. Two coarse reorientation algorithms are considered, each one may be implemented right after the separation from the launch vehicle without initial detumbling. First integrals of averaged equations are obtained in case the necessary axis of symmetry attitude coincides with the axis of the averaged geomagnetic field model circular cone. It is shown that implementation of the spinning algorithm with the nutation damping at the same time leads to the spinning of the satellite without increase of equatorial component of angular velocity. In the fine reorientation mode the solution of the equations of motion is obtained in terms of explicit formulas for the same special case. For the perpendicular direction the solution in quadratures is obtained.

For all algorithms the dependence of the time-response with respect to the orbit inclination is studied and recommendation what values of inclination are privileged is given.

7. Acknowledgements

The work is supported by the RFBR (grants №№ 09-01-00431 and 07-01-92001).

8. References

1. Likins P.W. Attitude stability criteria for dual spin spacecraft // Journal of Spacecraft and Rockets. 1967. V. 4, № 12. p. 1638–1643.

2. Shigehara M. Geomagnetic attitude control of an axisymmetric spinning satellite // Journal of Spacecraft and Rockets. 1972. V. 9, № 6. p. 391–398.

3. Renard M.L. Command laws for magnetic attitude control of spin-stabilized earth satellites // Journal of Spacecraft and Rockets. 1967. V. 4, № 2. p. 156–163.

4. Artuhin Y.P., Kargu L.I., Simaev V.L. Spin-stabilized satellites control systems. Moscow: Nauka, 1979 (in Russian).

5. Shrivastava S.K., Modi V.J. Satellite attitude dynamics and control in the presence of environmental torques – a brief survey // Journal of Guidance, Control, and Dynamics. 1983. V. 6, № 6. p. 461–471.

6. Grebenikov E.A. Averaging in Applied Problems. Moscow: Nauka, 1986. 256 p. (in Russian).

7. Beletsky V.V., Novogrebelsky A.B. Occurence of Stable Relative Equilibrium of a Satellite in Model Magnetic Field // Astronomical Journal. 1973. V. 50, № 2. p. 327–335 (in Russian).

8. Beletsky V.V. Motion of a Satellite about its Center of Mass in the Gravitational Field. Moscow: MSU publishers, 1975. 308 p. (in Russian).

9. Beletsky V.V., Khentov A.A. Tumbling Motion of a Magnetized Satellite. Moscow: Nauka, 1985. 288 p. (in Russian).

10. Bulgakov B.V. Applied Gyrostat Theory. Moscow: Gostehizdat, 1939. 258 p. (in Russian).

11. Beletsky V.V. Evolution of a Rotation of an Axisymmetric Satellite // Kosm. Issl. 1963. V. 1, № 3. p. 339–385 (in Russian).

12. Chernousko F.L. On a Motion of a Satellite About Its Center of Mass in a Gravitational Field // Prikl. Matem. i Mekh. 1963. V. 27, № 3. p. 473–483 (in Russian).

13. E.T. Wittaker, A Treatise on the Analytical Dynamics of Particles and Rigid Bodies, Cambridge University Press, 1988.

14. Hur P.S., Melton R.G., Spencer D.B. Meeting Science Requirements for Attitude Determination and Control in a Low-power, Spinning satellite // Journal of Aerospace Engineering, Sciences and Applications. 2008. V. 1, № 1. p. 25–33.

23

Contents