International Astronautical Forum
20th Anniversary of Astronautical School
Harbin, China – June 7-10, 2007

SPACE FLIGHT OF GEOSTATIONARY SATELLITE TO THE EARTH WITH LUNAR GRAVITY ASSIST

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Dear Colleagues!

First of all, I'd like to congratulate the Harbin Institute of Technology with the 20th Anniversary of your School of Astronautics and to give you the Congratulations of our Keldysh Institute of Applied Mathematics with our best wishes of your health and new successes in Astronautics.

Now let me to present you my study.

The Study deals with a problem of the spacecraft (SC) re-entry from Geostationary Equatorial Orbit (GEO) to the Earth. In addition to "direct" return trajectories with initial decreasing velocity of spacecraft, there are proposed, received and studied some "detour" ones using initial increasing velocity, flight to the Moon with a lunar gravity assist and following flight to the Earth.

An «exact» numerical analysis and a qualitative theoretical one are performed for this problem.

Conditions for realization of this flight are found.
It is shown that from the energy point of view the "detour" scheme with the lunar gravity assist is better considerably than the "direct" one.

The CONTENTS of the Presentation is given below.

CONTENTS

I. INTRODUCTION - LAUNCH OF SPACECRAFT TO GEOSTATIONARY ORBIT

First, about the problem of the SC launch to GSO. Two shemes of the SC launch to GEO from LEO are analysed [V.V. Ivashkin, N.N. Tupitsyn, 1970]

Fig. 1 gives their ΔV- comparison. Here:

w_f^I is the summary ΔV for two- or three-impulse launch in the Earth gravitation field;

w_f^{II} is the summary ΔV for detour two-impulse launch in the Earth-Moon-Sun gravitation field with a lunar gravity assist. This scheme is optimal for initial inclination i_1>~28°.

Fig. 2 gives this detour trajectory.

Fig. 3 gives the trajectory for this launch of the SC ASIA SAT 3/HGS-1 to GEO, 1998.

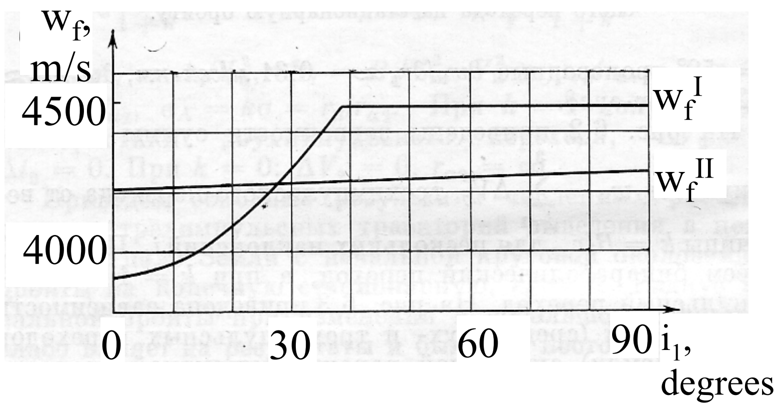


Fig. 1: ΔV- comparison of two schemes for the SC launch to GEO

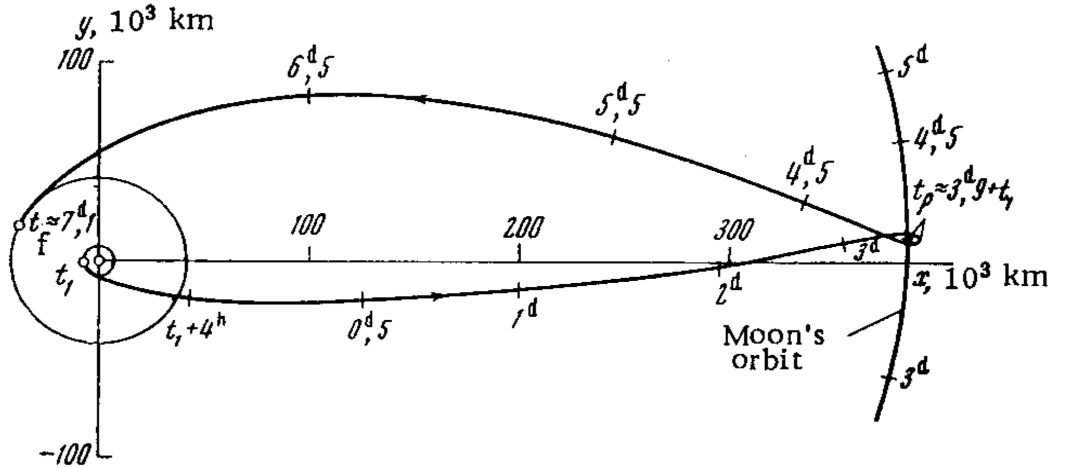


Fig. 2: Detour trajectory of the SC launch to the GEO with lunar gravity assist [V.V. Ivashkin, N.N. Tupitsyn, 1970,1971]

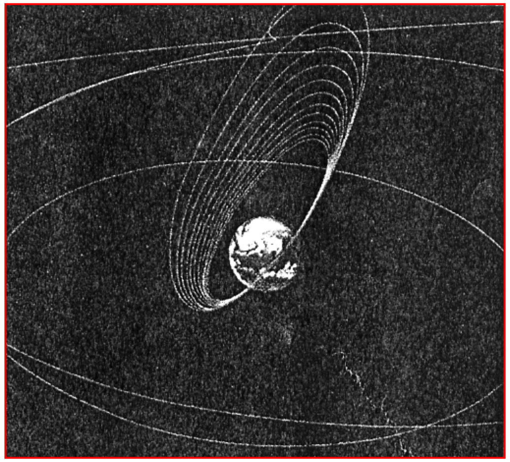


Fig. 3: Trajectory for a flight of SC ASIA SAT 3/HGS-1 to GEO [Riebe T., Schweitzer M., 1998]

2. "DIRECT" RE-ENTRY OF SPACECRAFT FROM GEO TO EARTH

A) Importance for the problem of the SC removal from the GEO.
Present removal of SC from GEO is not the best solution of this problem.
The SC re-entry from GEO to Earth for the SC destruction or its landing on the Earth surface is an option to give better solution.

B) Two schemes of re-entry are analysed: direct re-entry and detour one.

C)Fig. 4 gives **a scheme of "direct" re-entry from GEO to Earth with initial decreasing the SC velocity at $\Delta V^{(I)}$. A radius** $r_{\pi f}$ **in a final osculating perigee is a** ***variable parameter*** of the problem**:**

$$0 \le r_{\pi f} \le r_{\pi f\,\max} \approx 6421 \text{ km.}$$

Fig. 5 gives **a value of this velocity impulse value** versus the final radius $r_{\pi f}$**:**

$$\Delta V^{(I)} \approx 1.49\text{-}3.075 \text{ km/s.}$$

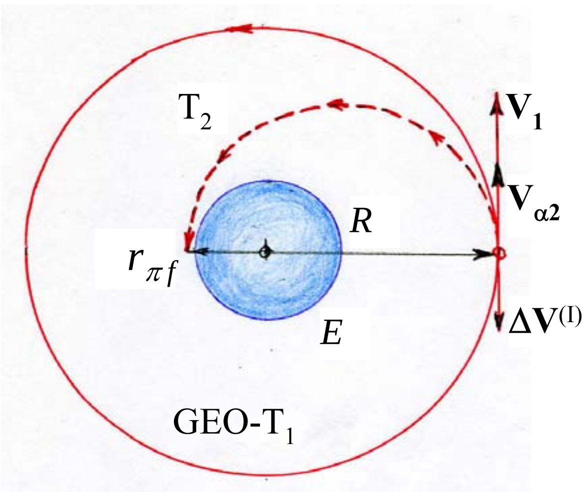


Fig. 4: "Direct" re-entry of SC from GEO to Earth.

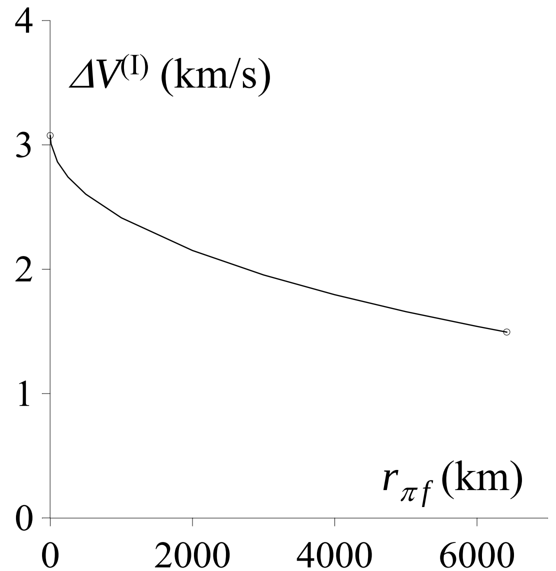


Fig. 5: ΔV for "direct" re-entry.

3. "DETOUR" FLIGHT FROM GEO TO EARTH WITH LUNAR GRAVITY ASSIST NUMERICAL ANALYSIS

a) "Detour" GEO-Earth flight is reverse to the Earth-GEO "detour" flight. It supposes increasing the SC velocity; after that the SC flies to the Moon to reach its vicinity, to perform a special lunar gravity assist and then to fly to the Earth atmosphere.

b) A numerical algorithm to determine detour trajectories is developed. It uses integration of the equations for a particle motion in the gravity field of the Earth (with its main harmonic c_{20}), the Moon, and the Sun.

Fig. 6 (geo-picture), and 7 (seleno-picture) give a typical detour trajectory. For it: accelerating velocity impulse $\Delta V^{(II)} \approx 1100$ m/s; - minimum distance to the Moon $\rho_\pi \approx 13{,}000$ km (~2/I-2001);- final perigee radius $r_{\pi f} \approx 6421$ km; - flight time $\Delta t_\Sigma \approx 9.4$ days.

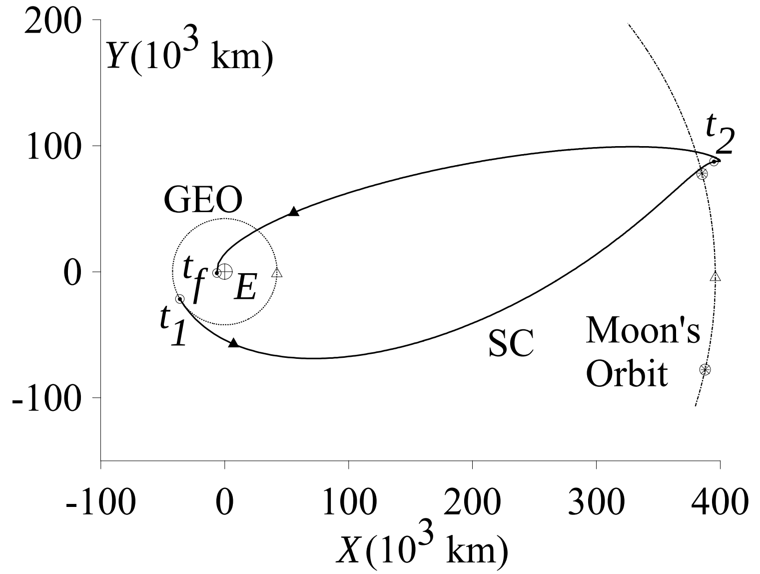


Fig. 6: The XY geocentric view for the GEO-Earth trajectory of "detour" type with lunar gravity assist

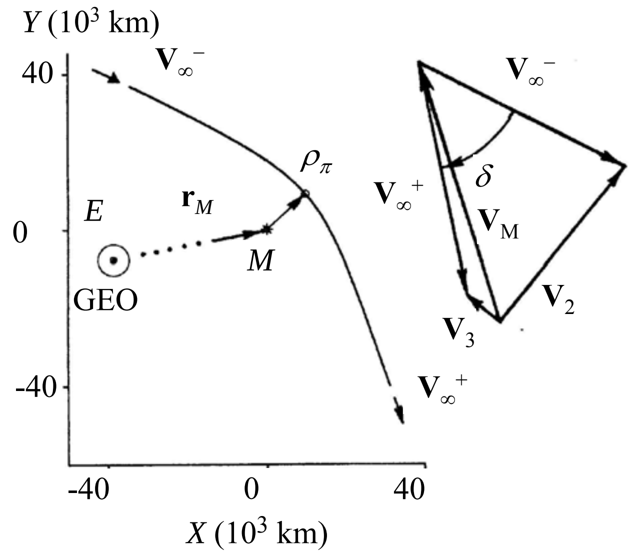


Fig. 7: The XY selenocentric view of the SC motion during the Moon encounter, and the SC velocity vector geometry for lunar gravity assist.

4. QUALITATIVE ANALYSIS OF GEO-EARTH "DETOUR" FLIGHT - a

A model of a "point" sphere of Moon's influence is used for the qualitative analysis of detour flight. For this model:

1. **Moon's sphere of influence is tightened into a point, and geocentric trajectory of the SC for the flight from initial orbit T_1 (GEO) to the Earth is presented by two conic arcs T_2 and T_3 that are connected at the Moon center for its flyby time t_2.**
2. **The Moon during its flyby must be at ascending or descending node of its orbit concerning the Earth equator.**
3. **Here, the geocentric velocity vector of the SC is changed from V_2 to V_3 according to the revolution of the selenocentric velocity at "infinity" to the angle δ - from V_∞^- к V_∞^+, see Fig. 8:**
   $V_\infty^- = V_2 - V_M$; V_M **is the Moon velocity.**
   $V_3 = V_M + V_\infty^+$;
   $|V_\infty^+| = |V_\infty^-| = V_\infty$; $V_3 \subset$**sphere** Σ, **see on next slide.**

4. **Velocity V_∞^+ is defined by the perilune radius ρ_π that is restricted by a condition:**

$$\rho_\pi \geq \rho_{\pi\,\min} \equiv R_M + \Delta R,$$

here R_M (=1738 km): Moon's radius,
ΔR (≈100 km): an error of the control system.

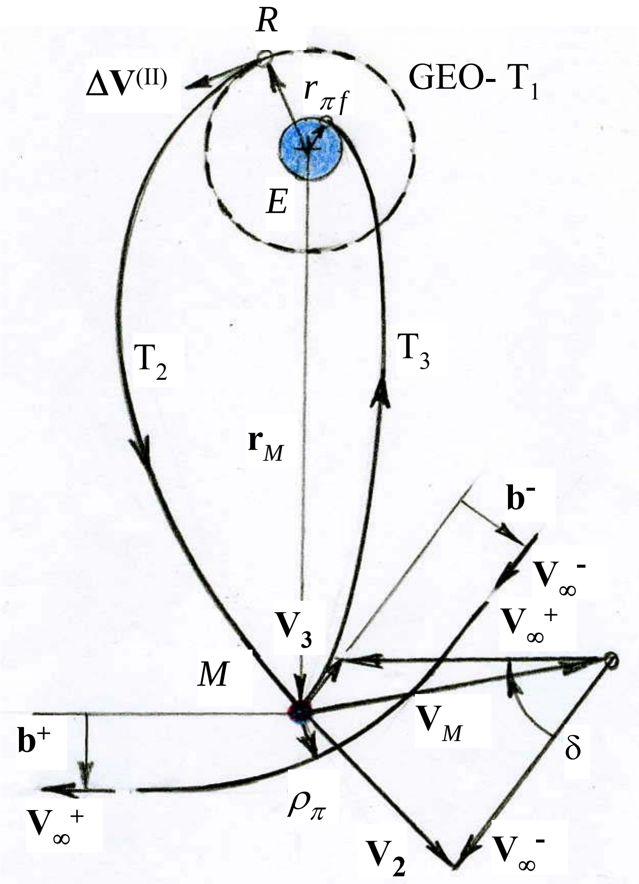


Fig. 8: Scheme of the SC flight from GEO to Earth in a model of a point sphere of Moon's influence.

4. QUOLITATIVE ANALYSIS OF GEO-EARTH "DETOUR" FLIGHT - b

Main parameters for the GEO-Earth detour trajectory

a) Radius $r_{\pi f}$ in the final osculating perigee of the orbit T_3 for the Moon-Earth flight:

$$0 \le r_{\pi f} \le r_{\pi\ max} \approx 6421\ \text{km};$$

$r_{\pi f}$: $V_3 \subset$ hyperboloid $H = \{ V_3 : r_{\pi 3}(V_3) = r_{\pi f} \}$; see Fig. 9.

b) apogee radius $r_{\alpha 2}$ for the GEO-Moon orbit T_2, or selenocentric velocity at "infinity" V_∞ :

$$r_{\alpha 2} \ge r_{\alpha 2\ min};\ V_\infty \ge V_{\infty\ min}(r_{\pi f});$$

$V_\infty(r_{\alpha 2})$: $V_3 \subset$sphere $\Sigma = \{ V_3 = V_M + V_\infty^+; |V_\infty^+| = V_\infty \}$;

$r_{\pi f}$, $r_{\alpha 2}$: *Solution* $V_3 \subset$intersection $A = H \cap \Sigma$, see Fig. 9;

If $V_\infty = V_{\infty\ min}$, there is one solution, the set A is a point.

If $V_\infty > V_{\infty\ min}$, A is a closed curve (or two ones), see Fig.9.

c) a parameter on the set A : Moon-Earth flight time or radial component V_{3r} of geocentric velocity V_3 near Moon for flight to Earth. If A is one curve, then:

$$V_{3r\ min}(r_{\pi f}, V_\infty) \le V_{3r} \le V_{3r\ max}(r_{\pi f}, V_\infty).$$

$V_{3r\ min} < V_{3r} < V_{3r\ max}$: two solutions $V_{3(1)}$ and $V_{3(2)}$, Fig. 9a.

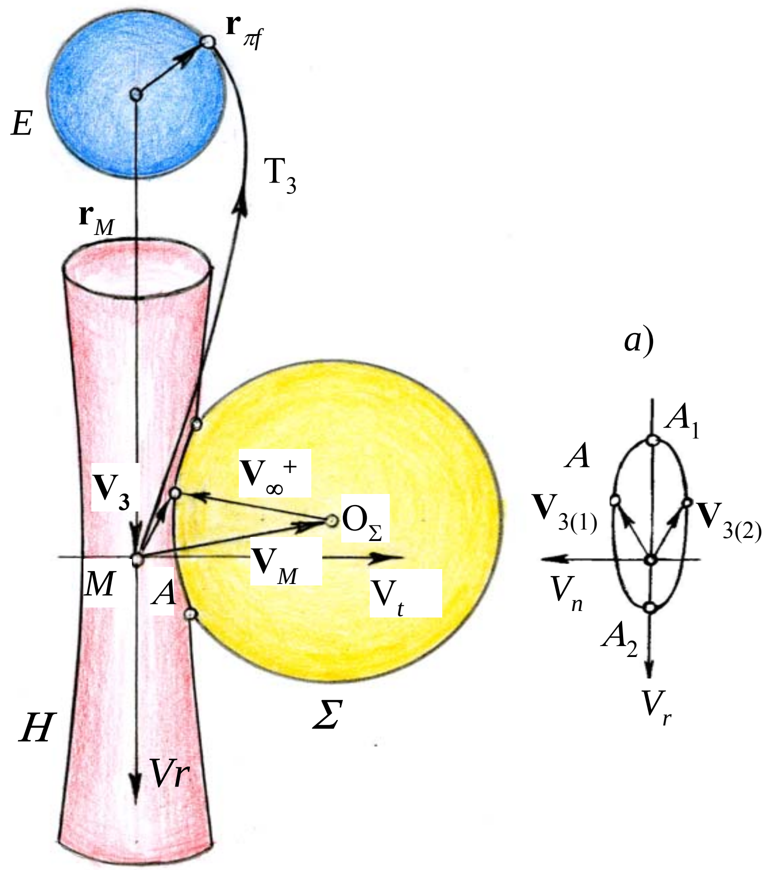


Fig. 9: Determination of the SC geocentric velocity V_3 and the trajectory T_3 for the flight to the Earth; a) View from the V_t axis to the set $A = H \cap \Sigma$.

4. QUOLITATIVE ANALYSIS OF GEO-EARTH "DETOUR" FLIGHT - c

Figure 10 gives some main characteristics of the SC detour re-entry from GEO to Earth versus geocentric distance at the final osculating perigee.

Solid lines correspond to the case of minimum values $V_\infty = V_{\infty\,\min}$, $r_{\alpha 2} = r_{\alpha 2\,\min}$

The velocity impulse $\Delta V^{(II)}$ for the detour scheme is **less essentially** than that $\Delta V^{(I)}$ for the direct re-entry: $\Delta V^{(II)} \approx 1.1$ km/s, $\Delta V^{(I)} - \Delta V^{(II)} \approx (0.4\text{-}1.9)$ km/s.

The SC **does not collide** with the Moon surface, $\rho_\pi > \rho_{\pi\,\min} \approx 2\,000$ km.

Minimum radius for apogee of the GEO-Moon orbit T_2: $r_{\alpha 2\,\min} \approx (450\text{-}570) \cdot 10^3$ km.

The dot-and-dash lines correspond to a case $r_{\alpha 2} > r_{\alpha 2\,\min}$, $V_\infty > V_{\infty\,\min}$

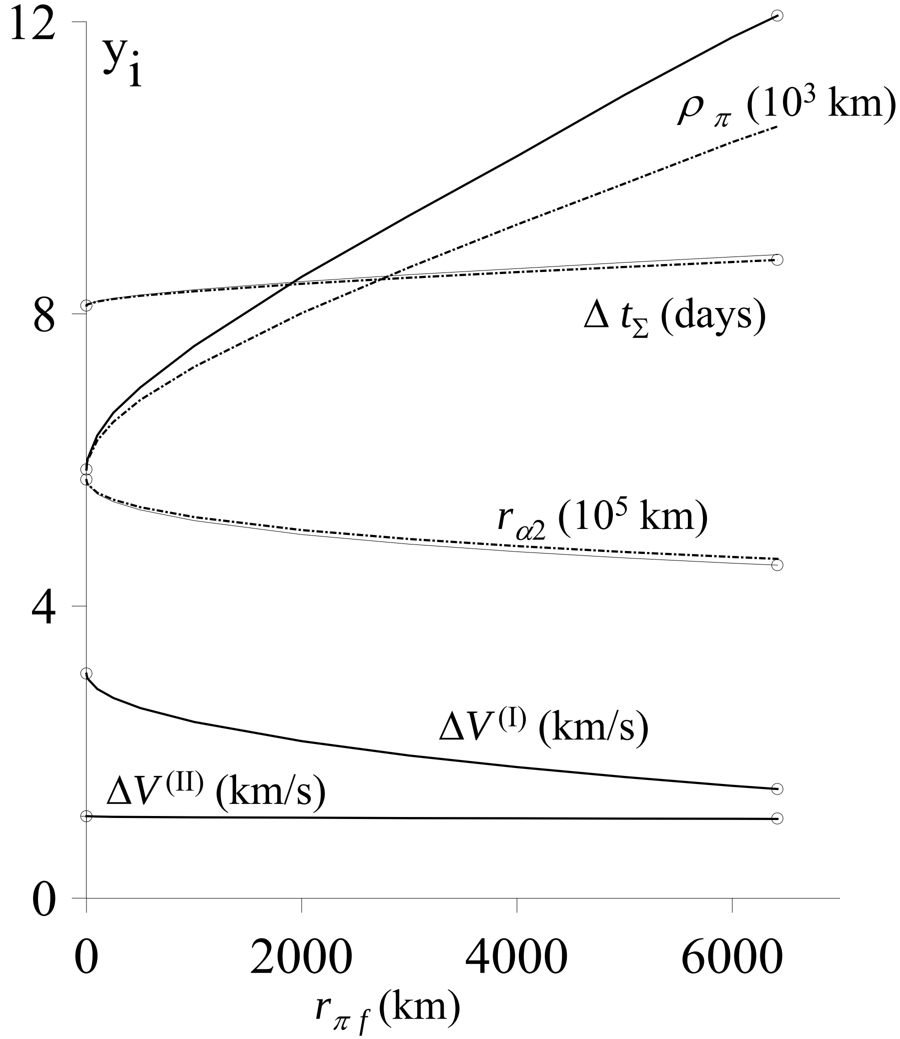


Fig. 10: Characteristics of the SC re-entry from GEO to Earth versus geocentric distance at the final osculating perigee (fly-by the Moon at January 2, 2001).

4. QUOLITATIVE ANALYSIS OF GEO-EARTH "DETOUR" FLIGHT - d

Let the values $r_{\pi f}$ **and** $r_{\alpha 2}$ **be fixed,** then

Figure 11 gives characteristics of trajectories for **the set A (a closed curve)**:

- inclination i_3 of the Moon-Earth orbit $\mathbf{T_3}$;
- angle δ for the revolution of the selenocentric velocity at "infinity" V_∞ during the lunar gravity assist;
- time of flight Δt_Σ;
- Minimum distance to Moon $\rho_{\pi\,min}$(~2,000 km);
- perilune radius ρ_π(it is more then $\rho_{\pi\,min}$).

Remark. Angles i_3 , δ and radii ρ_π differ for both solutions $V_{3(1)}$ and $V_{3(2)}$.
But the times Δt_Σ are equal.

Theorem: ***If the perigee radius $r_{\pi f}$ and radial geocentric velocity component V_{3r}, are fixed then all the solutions of the problem have the same parameters in the orbit plane (semi-major axes, apogee radius, true anomaly, the flight time) and differ by the orbit orientation in space only.***

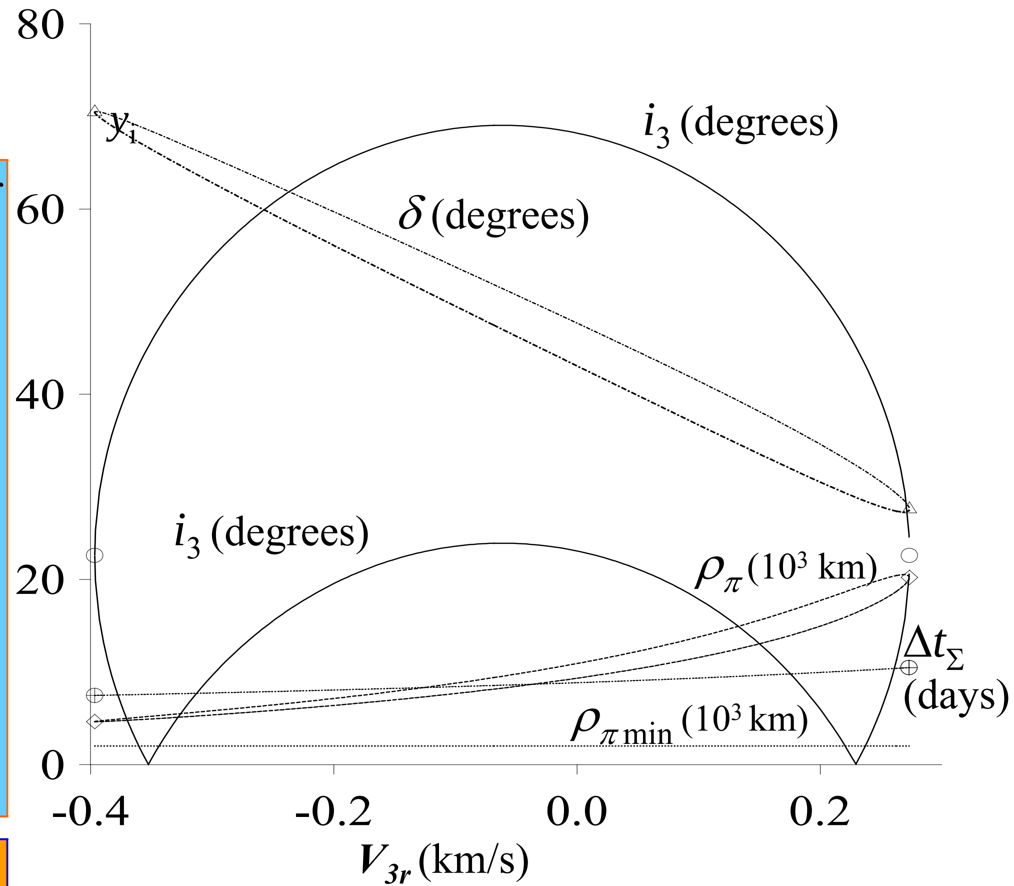
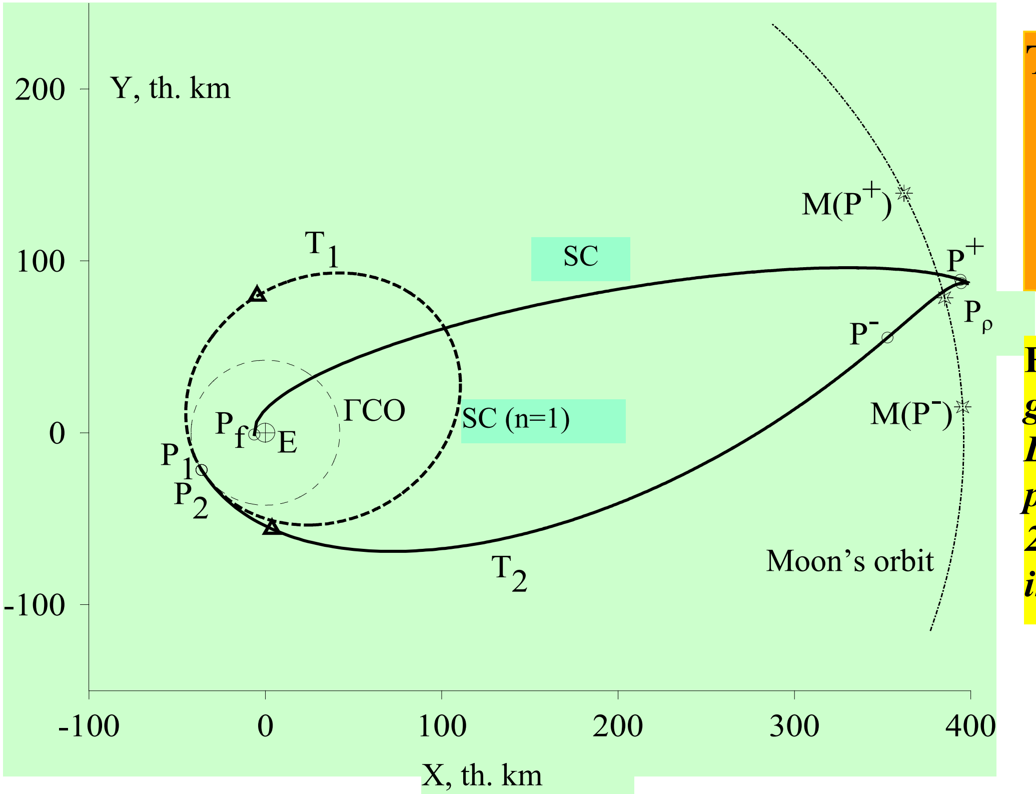


Fig. 11: Characteristics of the GEO-Earth detour flight versus the radial velocity component for the fixed values $r_{\pi f}$ and $r_{\alpha 2}$ ($r_{\pi f}$=6,421 km; $r_{\alpha 2}$= 490,000 km)

5. REMARKS - a

A transfer from an initial point on the GEO when a geographic longitude is given for this point can be provided for the detour GEO-Moon-Earth flight with various methods.

E.g., this can be made using two- (or more) impulses start of the SC from GEO to the lunar orbit and adding one (or several) passive orbital revolutions between them, see Fig. 12 (for "soft" re-entry, $r_{\pi f}$=6,421 km) and Fig. 13 (for "hard" re-entry, $r_{\pi f}$=0).



This will also decrease the gravitational losses of the velocity and the fuel consumption during the engine operation for the SC start to the Moon (Robbins H.M.).

Figure 12. ***"Soft" re-entry of GSS with fixed geographical longitude from GSO to Earth with Lunar Gravity Assist: t_0=27.12.2000 – P_1; a parking revolution on T_1; start to Moon on 29.12.2000 – P_2; arrival to Earth on 8.01.2001; initial longitude λ_0≈16°.***

5. REMARKS - b

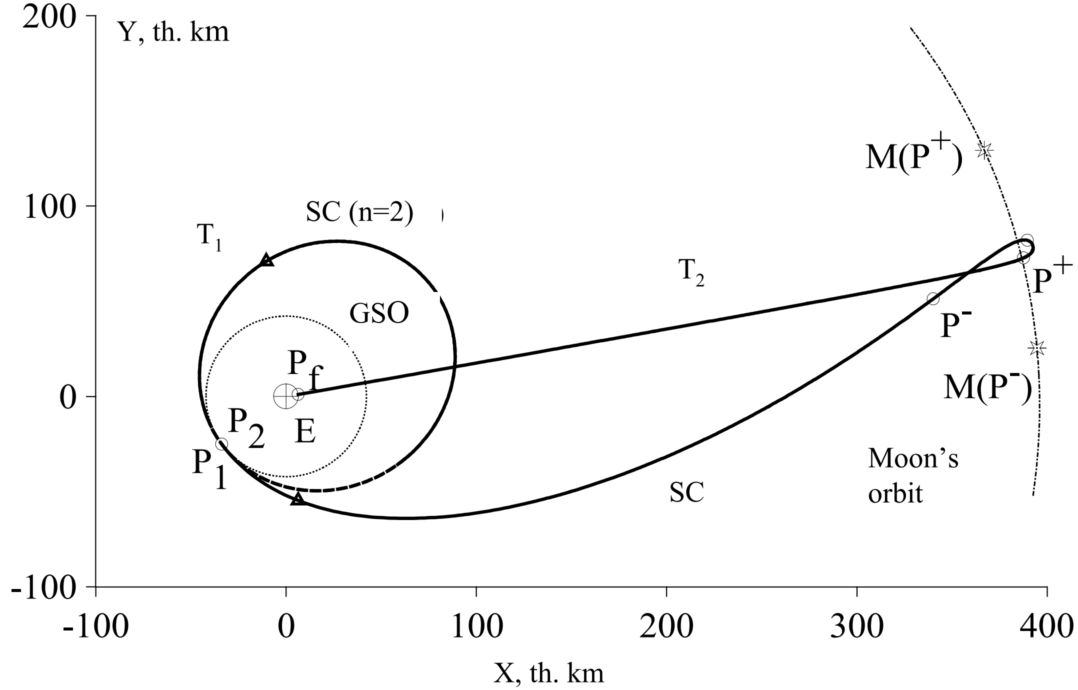


Figure 13. ***"Hard" re-entry of GSS with fixed geographical longitude from GSO to Earth with Lunar Gravity Assist: t_0=26.12.2000 – P_1; two parking revolutions on T_1; start to Moon on 30.12.2000 – P_2; arrival to Earth on 7.01.2001; initial longitude $\lambda_0 \approx 16°$.***

6. CONCLUSIONS

Analysis performed has shown that using the lunar gravity assist allows realization of the detour low-energy trajectories for the GEO-Earth re-entry with passive decreasing the perigee radius of the SC orbit.

Due to this, one can do this re-entry essentially better from energy point of view, with the smaller fuel consumption than the usual “direct” flight.

Of course, this scheme requires the larger duration of the flight and more exact navigation and control systems.

In Conclusions, I’m glad once more to congratulate the Harbin Institute of Technology with the 20^{th} Anniversary of your School of Astronautics and to wish you the best.

The author thanks the Harbin Institute of Technology for the invitation to take part in this International Astronautical Forum. The author is grateful to Prof. Liu Dun for the help in my staying in the China and translating my Presentation to Chinese.

The study is supported by the Russian Foundation of the Basic Studies (Grant N 06-01-00531) and by the Grant for Scientific School NSh-2448.2006.1.

Thank you for your attention !

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