MODELING OF ROTATORY FLOWS IN THE PLASMA ACCELERATOR CHANNEL WITH LONGITUDINAL MAGNETIC FIELD

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The numerical model of two-dimensional flows in coaxial channels of plasma accelerators is considered in presence of the longitudinal magnetic field. Computations are based on the MHD-equations taking into account the Hall effect, electrical conductivity tensor. The analytical and numerical solutions are analyzed and compared with reference to steady-state plasma flow in case of the ion current transport.

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INTRODUCTION

Many factors, above all the current crisis [1] due to the Hall effect, impede the achievement of high velocities in plasma accelerators. In order to overcome the negative consequence of the Hall effect it is appropriate to go to systems of the quasi-steady plasma accelerator type proposed in [2]. The experiments generally confirmed the underlying ideas on the basis of which such plasma accelerators are designed. The experimental investigations [3-5] show a real opportunity to get flows of relatively dense plasma $n \geq 10^{14} cm^{-3}$ with $V \approx 10^6 \div 10^8\ cm/c$. Such opportunities allow using plasma accelerators in space as the electric jets and in various applications including thermonuclear installations as well.

The mathematical models [6,7] of the plasma-dynamics play an important role accelerator design. For dense plasma the processes in accelerators are investigated, theoretically and numerically, within the framework of the MHD equations.

The presence of a longitudinal magnetic field $\left(H_\varphi << H_z << H_r\right)$ opens up new possibilities for controlling the dynamic processes in the accelerator channel and makes to realize transonic flow in a channel of certain geometry.

In [8] the effect of a longitudinal magnetic field on two-dimensional axisymmetric two-component plasma flows is determined analytically. The investigations are carried out within the framework of the smooth channel approximation for ideal magnetohydrodynamic equations. The longitudinal magnetic field complicates the flow. For example, it leads to plasma rotation about the axis of the system. In this case, an analysis of the most impotent properties of plasma flows showed that the Hall effect and the anode flow zone can be significantly reduced owing to the longitudinal field.

BASIC EQUATIONS

The numerical model of axial-symmetrical flow in plasma accelerator channels in the presence of a longitudinal magnetic field is elaborated. Computations are based on the MHD-equations taking into account the Hall effect ($\mathbf{V}_e \neq \mathbf{V}_i$), electrical conductivity tensor and transport coefficients in magnetic field depending on the $\omega_e \tau_e$ [9].

$$\frac{\partial \rho}{\partial t} + div \rho \mathbf{V} = 0\ ; \qquad \rho \frac{d\mathbf{V}}{dt} + \nabla P = \frac{1}{c}\left[\mathbf{j}, \mathbf{H}\right]$$

$$\rho \frac{d\varepsilon}{dt} + P\ div\,\mathbf{V} = Q - div\,\mathbf{q} + \frac{k}{e(\gamma-1)}\left(\mathbf{j}, \nabla\right) T + \frac{P_e}{e}\, div \frac{\mathbf{j}}{n}$$

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -rot\,\mathbf{E}\ ; \quad \mathbf{E} = -\frac{1}{c}\left[\mathbf{V}_e\ ,\ \mathbf{H}\right] - \frac{1}{en}\nabla P_e + \frac{1}{en}\mathbf{R}$$

$$\mathbf{j} = \frac{c}{4\pi} rot\,\mathbf{H} = e\,n\left(\mathbf{V}_i - \mathbf{V}_e\right)\ ; \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \left(\mathbf{V}, \nabla\right)$$

$$P = P_i + P_e = 2\left(c_p - c_v\right)\rho T\ ; \qquad \varepsilon = 2\,c_v\,T$$

Here $\mathbf{V} = \mathbf{V}_i$; $n_i = n_e = n$; $T_i = T_e = T$; P - total pressure, $\rho = m\,n$ - density of heavy particles, $\mathbf{j}$ - electric current; $\mathbf{q}$ - heat flux.

$$\mathbf{R} = \mathbf{R}_\mathbf{j} + \mathbf{R}_T\ ; \qquad Q = Q_i + Q_e = \frac{1}{en}\left(\mathbf{R}, \mathbf{j}\right)$$

$$\mathbf{R}_\mathbf{j} = \frac{en}{\sigma}\left(A_1(x)\,\mathbf{j}_{//} + A_2(x)\,\mathbf{j}_\perp + \frac{A_3(x)}{H}\left[\mathbf{H}, \mathbf{j}\right]\right)$$

$$\mathbf{R}_T = -k\,n\left(B_1(x)\nabla_{//}T + B_2(x)\nabla_\perp T - \frac{B_3(x)}{H}\left[\mathbf{H}, \nabla T\right]\right)$$

$$\mathbf{j}_{//} = \frac{1}{H^2}\left(\mathbf{j}, \mathbf{H}\right)\mathbf{H}\ ; \qquad \mathbf{j}_\perp = \frac{1}{H^2}\left[\mathbf{H}, \left[\mathbf{j}, \mathbf{H}\right]\right]$$

$$\mathbf{q} = \mathbf{q}_e + \mathbf{q}_i \approx \mathbf{q}_\mathbf{j}^e = -\frac{kT}{e}\left(B_2(x)\,\mathbf{j} + \right.$$
$$\left. + \frac{B_1(x) - B_2(x)}{H}\left(\mathbf{j}, \mathbf{H}\right)\mathbf{H} + \frac{B_3(x)}{H}\left[\mathbf{H}, \mathbf{j}\right]\right)$$

$$\sigma = \frac{e^2\,n_e\,\tau_e}{m_e} \sim T^{3/2}$$ - conductivity of medium .

$A_{1,2,3}(x)$ and $B_{1,2,3}(x)$ are known functions of

$$x = \omega_e\,\tau_e, \qquad \omega_e = \frac{eH}{m_e c}, \quad \tau_e = \frac{3\sqrt{m_e}\,(kT)^{3/2}}{4\sqrt{2\pi}\,\lambda\,e^4\,Z^2\,n}$$

In the case of axial flow symmetry $\left(\partial/\partial\varphi = 0\right)$ it is possible to introduce the vector potential $\mathbf{A}$ $\left(\mathbf{H} = rot\,\mathbf{A}\right)$ so that $div\,\mathbf{H} = 0$ in the numerical simulations. The toroidal vector potential A_φ defines the poloidal components of the magnetic field H_z and H_r .

As a result, we have 7 equations for variables $\rho,\ T,\ V_z,\ V_r,\ V_\varphi,\ H_\varphi,\ A_\varphi$.

PARAMETERS, BOUNDARY CONDITIONS AND NUMERICAL METHODS

As the input dimensional units, we will take the characteristic values of the density and temperature $\rho_o = m\, n_o$ and $T_o = T_o^e = T_o^i$ at the channel inlet, the length L of the plasma accelerator channel and the strength $H_o = H_o^\varphi = \frac{2\, J_p}{c\, R_o}$ of the azimuthal magnetic field component, where J_p is the current in the system and R_o is the characteristic radius at the inlet. Using these quantities, we form the following units of velocity, pressure and time scales: the characteristic Alfven velocity $V_o = H_o / \sqrt{4\,\pi\,\rho_o}$, $H_o / 4\,\pi$ and L / V_o, respectively. In this case the dimensionless parameters in numerical investigation have the form:

$$\beta = \frac{8\,\pi\,P_o}{H_o^{\,2}}\ (P_o = k\, n_o\, T_o); \qquad \xi = \frac{c}{e\,L}\sqrt{\frac{m_i}{4\,\pi\,n_o}}$$

$$\nu = \frac{1}{\mathrm{Re}_m} = \frac{c^2}{4\,\pi\,L\,V_o\sigma}\ ; \qquad \omega_e\,\tau_e = \frac{\xi\,H}{\nu\,\rho}$$

The dimensionless conductivity $\mathrm{Re}_m = \sigma_o T^{3/2}$ (magnetic Reynolds number) contains σ_o which can be expressed in terms of the initial dimensional parameters and physical constant.

The boundary condition at the channel inlet corresponds to the subsonic plasma inflow with $\rho(r) = f_1(r)$; $T(r) = f_2(r)$; $V_\varphi = 0$; $H_z(r) = H_z^o$, where $f_1(r)$ and $f_2(r)$ are known functions of r. We will assume that the total electric current flowing through the system is supplied only through the electrodes and maintained constant. This generates the boundary condition at the inlet for azimuthal magnetic field $r\,H_\varphi = r_o = R_o / L$.

The accelerator plasma dynamics are investigated in different current transport regimes. In the electron current transport regime the streamlines of the ion plasma component lie on the impermeable electrode surfaces. In this case the electrodes are not equipotential. On the other hand, in the ion current transport regime the electrodes are equipotential surfaces. In this case they must be transparent for the plasma entering the channel across them. Most experiments [3-5] and models [6,8,10] are based on ion current transport.

In the present study we will consider cold plasma flow in the ion current transport regime. In this case we have $E_\tau = 0$ on the electrode surfaces: the cathode $r_k(z)$ and the anode $r_a(z)$. The shapes of the electrodes are given. In the presence of a longitudinal magnetic field it is necessary to determine the additional condition for magnetic field components H_z and H_r. Within the framework of simple formulation of the problem it is natural to assume that $H_n = 0$ on the electrode surfaces. Besides we will suppose that $\partial T / \partial n = 0$. The anode flow enters the channel from the side of the anode. Therefore, on the anode surface we must assign two functions $\rho(z) = f_3(z)$ and $V_\varphi = f_4(z)$. The functions $f_1(r),\ f_2(r),\ f_3(z),\ f_4(z)$ must be chosen depending on the particular details of the problem formulation. These functions can be determined from analytical model in order to compare the analytical and numerical solutions.

The channel geometry is specified by the electrode profiles, which correspond to the possibility of transonic flow so that at mid-channel the flow velocity passes through the local velocity of the fast magnetosonic wave. So at the accelerator outlet the boundary conditions correspond to a supersonic plasma flow.

The simulations are based on the adaptation of flux-corrected transport method for hyperbolic part of equations. The finite conductivity is taken into account in the parabolic equations for A_φ and H_φ fields by means of the implicit numerical scheme. A steady-state supersonic flow in the nozzle-type channel calculated by the relaxation method.

RESULTS OF COMPUTATION

The channel has the shape of a Laval nozzle of length equal to unity. The initial dimensional and dimensionless parameters are: $n_o = 3.6 \cdot 10^{14} cm^{-3}$; $T_o = 2$ эВ; $L = 60\ cm$; $J_p = 300\ kA$; $\beta = 0.005$; $\xi = 0.02$; $\sigma_o = 812.8$. For hydrogen plasma we have $\nu = 0.0012$ and $\omega_e\,\tau_e = 16.2$ if $T = 1$ and $\rho = 1$. Within the framework of the model proposed the presence of a small insignificant longitudinal magnetic field ($H_z^o = 0.1$) makes it possible to realize transonic flow.

In Fig.1 we have reproduced the transonic flow in the ion current transport regime. In Fig. 1-c the length of the vectors (in centimeters) is equal to the dimensionless value of the velocity at a given point. At mid-channel the flow velocity passes through the local velocity of the fast magnetosonic wave. In Fig. 1-b the curves show the level lines of the function V_φ characterizing the rotation for $H_z^o \neq 0$. The azimuthal velocities have maxim in the neighborhood of the anode closer to the acceleration channel exit. Under certain conditions in the absence of a longitudinal field, a shortage of ions develops in this region due to the Hall effect. Current crisis and the collapse of the acceleration process accompany this. In accordance with the results of this investigation, in the presence of a longitudinal field the density on the anode increase due to ration.

The two-dimensional analytic and numerical solutions are analyzed and compared with reference to a plasma accelerator channel. Computer experiments describing the rotatory plasma flows in presence of a longitudinal magnetic field demonstrated the similarity of the results obtained on the basis of analytic model [10].

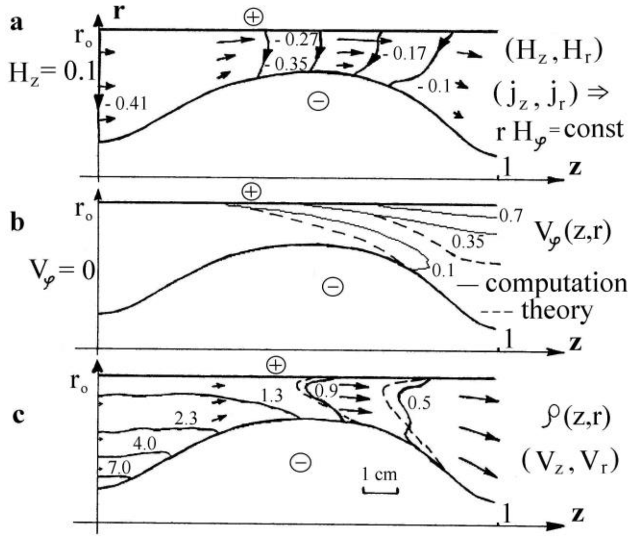

Fig. 1. Plasma flow in the presence of a longitudinal magnetic field: a) electric current (j_z, j_r) (level lines of the function $r H_\varphi = const$) and vector magnetic field distribution; b) level lines of the ion azimuthal velocity V_φ; c) density distribution $\rho(z, r)$ and vector velocity field of the ion component (V_z, V_r). The broken curve corresponds to the analytical model.

CONCLUSION

A computer simulation of rotatory axisymmetric steady-state plasma flows in the presence of a longitudinal magnetic field was carried out in terms of two-dimensional time-dependent MHD-equations, taking into account the Hall effect, electrical conductivity tensor and transport coefficient in magnetic field depending on the $\omega_e \tau_e$. A longitudinal magnetic field provides additional possibilities for controlling the processes in the channel of coaxial plasma accelerator and makes it possible to realize transonic flow with the ion current transport.

The calculation results correspond to the analytical model of rotatory plasma flow.

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МОДЕЛИРОВАНИЕ ВРАЩАЮЩИХСЯ ПОТОКОВ В КАНАЛЕ ПЛАЗМЕННОГО УСКОРИТЕЛЯ ПРИ НАЛИЧИИ ПРОДОЛЬНОГО МАГНИТНОГО ПОЛЯ

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Представлена численная модель двумерных течений в коаксиальных каналах плазменных ускорителей при наличии продольного поля. В основу модели положены МГД-уравнения с учетом эффекта Холла, тензора проводимости среды. Проведено сопоставление аналитического решения и результатов расчетов установившегося течения плазмы в режиме ионного токопереноса.

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