Network of stochastic coupled oscillators and one-way quantum computations

Eugene Grichuk, Margarita Kuzmina and Eduard Manykin

Abstract—**A model of qubit has been designed as a stochastic oscillator formed by system of two coupled limit cycle oscillators with randomly perturbed limit cycles and own frequencies. The oscillatory qubit imitates electrical field vector behavior of polarized light beam. It adequately simulates both pure and mixed states of two-level quantum system. A cluster of entangled qubits, that is usually exploited as a computation resource in one-way quantum computation schemes, is suggested to design as synchronized network of the oscillatory qubits. System of equations governing the oscillatory network evolution has been written. The example of one-qubit gate is constructed.**

I. INTRODUCTION

Quantum computations is the interdisciplinary research field undergoing active development Currently both quantum physicians and information theory experts focused their attention on theoretical analysis and experimental realizations of quantum computation algorithms. After the discovery of Shor's quantum algorithm for large number factorization it become clear that quantum algorithms are capable to provide an effective solution of some mathematical problems for which exist no effective classical algorithms. The development of quantum calculations has been stimulated the appearance of new research branches such as quantum informatics [2-4], arisen at the intersection of quantum physics and information theory.

Quantum computation algorithms are based on evolution of some quantum systems and exploitation of quantum physics laws for computation performance. In the way the type of parallelization is used that is inherent solely to quantum systems and cannot be realizable via traditional classical computers.

Modern quantum computation schemes are based on construction of sequences of unitary gates imposed into quantum network. One-qubit gates define modifications of single qubit states, whereas two-qubit gates specify the character of qubit interactions. In 2001 a significantly new type of quantum computation scheme has been proposed – so called one-way, or cluster quantum computations (CQC) [5-7]. The feature of one-way QC is that the sequence of measurements, necessary to readout the information from qubit cluster, is explicitly included in CQC computation scheme. So, each qubit cluster, initially prepared in maximally entangled state, undergoes irreversible evolution via one-qubit measurements in the process of computations. As a result the coherent cluster state is turned out to be inevitably destroyed by measurement process, and thus the cluster can be used for computations only once. The choice of measurement sequence just defines the quantum computation algorithm itself. The computation process can be regarded as a process of information transfer through system canals, similar to percolation phenomenon [5] (see fig. 1).

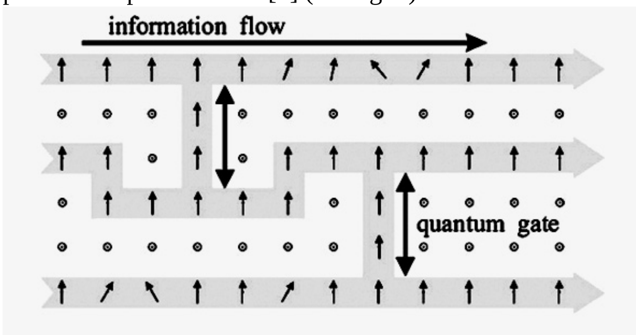

Fig. 1. Network scheme for one-way quantum computations (from [5])

One could even say that information processing in one-way quantum calculation schemes is really performed in a classical level, although quantum physical principles have been used in preparation of cluster of strongly entangled qubits. It was shown that, for instance, the CQC scheme is ideally suitable for realization of Grover's algorithm [2].

So, the problem of construction of one-way quantum computation schemes requires analysis of qubit cluster evolution at variable qubit entanglement degree. We try to develop a network approach to the problem. Namely, we suppose that at some restrictions to the problem it can be formulated in terms of state evolution of network of coupled stochastic oscillators under special sequence of external actions on single oscillators. Surely, the

This work was supported in part by the Mathematics Dept. RAS
Eugene Grichuk is a post graduate student of National Research Nuclear University "MEPI" (e-mail: es@t-25.ru);
Margarita Kuzmina is with Keldysh Institute of Applied Mathematics RAS, Miusskaya Sq.4, 125047 Moscow, Russia (phone: 7-499-972 3491; fax: 7-499-972 0737; e-mail: mg.kuzmina@gmail.com;
Eduard Manykin is with Russian Research Center "Kurchatov Institute", Kurchatov Sq. 1, 123182, 123182 Moscow, Russia (phohe: 7-495-196 9107; e-mail: emnany@isssph.kiae.ru .

preliminary design of proper oscillatory model of single qubit is necessary. The qubit model should be capable to adequately imitate the features of two-level quantum-mechanical system. The example of oscillatory model of qubit is just presented in the paper. Also the system of equations, governing dynamics of the designed oscillatory network is written, and the models for one-qubit gates are suggested.

II. BEAM OF QUASI-MONOCHROMATIC POLARIZED LIGHT AS A QUBIT. QUANTUM AND CLASSICAL LEVELS OF BEAM DESCRIPTION

As it is known, a qubit (quantum bit of information) can be described as a two-level quantum-mechanical system that can be either in pure or in mixed quantum state. So, qubit state should be generally understood as a state of statistical ensemble of identical quantum systems described by density operator (density matrix) $\hat{\rho}$ satisfying the conditions

$$\det \hat{\rho} \geq 0, \quad Tr\hat{\rho} = 1. \quad (1)$$

In the case of pure qubit state, defined by a column state function $\langle \psi |$, the density operator is reduced to one-dimensional projector onto the state $\langle \psi |$, $\hat{\rho}_\psi = |\psi\rangle\langle\psi|$. Here we used the ordinary notations: operator $A = |\psi\rangle\langle\varphi|$ of rank 1 acts on state $\langle\chi|$ by formula $A|\chi\rangle = |\psi\rangle\langle\varphi|\chi\rangle$, where $\langle\varphi|\psi\rangle$ is inner product. If one uses the basis $\{\vec{e}_x, \vec{e}_y\} = \{(1\ \ 0)^T, (0\ \ 1)^T\}$, it is convenient to introduce the basis of Pauli matrices in real space of Hermitian matrices and represent the density operator in the form

$$\hat{\rho} = \frac{1}{2}(\hat{I} + p_x\hat{\sigma}_x + p_y\hat{\sigma}_y + p_z\hat{\sigma}_z) = \frac{1}{2}\begin{pmatrix} 1+p_z & p_x - ip_y \\ p_x + ip_y & 1-p_z \end{pmatrix}, \quad (2)$$

where

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$p_x = Tr(\hat{\rho}\hat{\sigma}_x), \quad p_y = Tr(\hat{\rho}\hat{\sigma}_y), \quad p_z = Tr(\hat{\rho}\hat{\sigma}_z), \quad (3)$$

and $P = (p_x, p_y, p_z)$ are Stokes parameters. From the condition $\det\hat{\rho} \geq 0$ it follows the restriction on *P*: $p_x^2 + p_y^2 + p_z^2 \leq 1$. Pure states are characterized by the condition $p_x^2 + p_y^2 + p_z^2 = 1$ and form the Bloch sphere (which is known also as Poincare sphere in optics).

As it is also known, Stokes parameters are used for description of polarization of classical electromagnetic radiation in terms of intensity, degree of polarization, shape and orientation of the polarization ellipse. Moreover, a beam of quasi-monochromatic light can be equivalently described both at quantum mechanical level (as a photon ensemble) and at classical level, in frames of classical electromagnetic field theory. At quantum level of description a beam of quasi-monochromatic light is considered as photon beam, propagating in a direction specified by vector $\vec{k}$. It can be described as statistical ensemble of photons with moment $\vec{p} = (ch\omega / c)\vec{k}$ and polarization state defined by two-dimensional unit vector $\vec{e}$, located in the plane orthogonal to $\vec{k}$. Stokes parameters characterize the ensemble in a mixed state from the viewpoint of its representation by a superposition of two sub-ensembles in pure states with polarization vectors $\vec{e}_x$ and $\vec{e}_y$. In the case of coherent superposition of the sub-ensembles we have a beam of fully polarized photons, in the case of completely non-coherent sub-ensembles superposition – a beam of unpolarized photons, and in an intermediate case of partially coherent sub- ensembles superposition – a beam of partially polarized photons.

From the viewpoint of classical electrodynamics a beam of quasi-monochromatic light is plane quasi-monochromatic electromagnetic wave, specified by its propagation vector $\vec{k}$. Electrical field vector $\vec{E}(t)$ of the electromagnetic wave, that is located in the plane, orthogonal to $\vec{k}$ (electromagnetic wave transversality), can be written as

$$\vec{E}(t) = E_x e^{i\alpha} \cdot \vec{e}_x + E_y e^{i(\alpha+\delta)} \cdot \vec{e}_y, \quad (\vec{e}_x, \vec{k}) = (\vec{e}_y, \vec{k}) = 0$$

For adequate description of light beam polarization in terms of electrical field one should represent $\vec{E}(t)$ is as a two-dimensional stationary random function of time. Let $\bar{\vec{E}}(t)$ be the mean of random function $\vec{E}(t)$ and so $\tilde{\vec{E}}(t) = \vec{E}(t) - \bar{\vec{E}}(t)$ be the fluctuation of $\vec{E}(t)$. For stationary random functions the mean $\bar{\vec{E}}(t)$ coincides with the mean over time, $\langle\vec{E}(t)\rangle$, that is

$$\bar{\vec{E}}(t) = \langle\vec{E}(t)\rangle = \bar{\vec{E}}(t) \equiv \lim_{T\to\infty}(2T)^{-1}\int_{-T}^{T}\vec{E}(t)dt.$$

There is the following relation between the coherence matrix $\hat{J}$ of quasi-monochromatic light beam in the basis $\{\vec{e}_x, \vec{e}_y\}$,

$$\hat{J} = [J_{mn}] = \frac{1}{2}\begin{pmatrix} \langle E_x E_x^*\rangle & \langle E_x E_y^*\rangle \\ \langle E_y E_x^*\rangle & \langle E_y E_y^*\rangle \end{pmatrix} \quad (4)$$

and correlation matrix $\hat{D}$ of 2D random function $\vec{E}(t)$:

$$J_{mn} = D_{mn}(0) = \tilde{D}_{mn}(0) + \langle E_m\rangle\langle E_n\rangle, \\ \tilde{D}_{mn}(\tau) = \langle\tilde{E}_m(t+\tau)\tilde{E}_n^*(t+\tau)\rangle \quad (5)$$

In optics Stokes parameters, denoted as Q, U, V , are used for beam polarization characterization. In the basis $\{\vec{e}_x, \vec{e}_y\}$ there is the following relation between light beam intensity I, Stokes parameters and coherence matrix $\hat{J}$ (correlation matrix $\hat{D}$ of stationary random function $\vec{E}(t)$):

$$I = J_{11} + J_{22}, \quad Q = J_{11} - J_{22},$$
$$U = J_{12} - J_{21}, \quad V = -i(J_{12} - J_{21}), \tag{6}$$

or

$$I = \langle E_x \rangle^2 + \langle E_y \rangle^2, \quad Q = \langle E_x \rangle^2 - \langle E_y \rangle^2,$$
$$U = 2\langle E_x\, E_y \cos(\delta)\rangle, \quad V = 2\langle E_x\, E_y \sin(\delta)\rangle. \tag{7}$$

The Stokes parameters characterize form and orientation of polarization ellipse – the curve that the end of random vector $\vec{E}(t)$ traces out in some plane, orthogonal to vector $\vec{k}$, the direction of beam propagation (fig. 2):

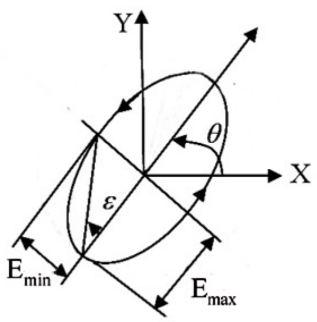

Fig. 2 Polarization ellipse

$$\theta = \frac{1}{2} arctg \frac{U}{Q}, \quad \varepsilon = \frac{1}{2} \arcsin(\frac{V}{(Q^2 + U^2 + V^2)^2},$$
$$E_{max}^2 + E_{min}^2 = Q^2 + U^2 + V^2. \tag{8}$$

The optical Stokes parameters are related to Stokes parameters p_x, p_y, p_z, defined in eq. (3), as

$$p_x = U / I, \quad p_y = V / I, \quad p_z = Q / I. \tag{9}$$

The inequality

$$Q^2 + U^2 + V^2 \le I^2 \tag{10}$$

is fulfilled for optical Stokes parameters (the equality takes place in the case of fully polarized light). The value $p = (Q^2 + U^2 + V^2)^{1/2} / I$ defines light polarization degree, that is, the part of fully polarized light in the beam.

Summarizing, we could say that a beam of quasi-monochromatic light, although being a macro-physical object, remains the necessary properties of two-level quantum system in its polarization characteristics. Therefore, it seems reasonable the attempt to design a proper qubit model relying on correlation theory of stationary random functions. It will allow us to further formulate some quantum computation problems in terms of controllable dynamics of artificial neural networks [11-13], qubit being network processing unit. In particular, one-way quantum computations seem to be naturally related to dynamical evolution of artificial feed-forward neural networks under gradual variation of both network processing unit states and network interconnection architecture.

III. QUBIT MODEL AS A SYSTEM OF TWO COUPLED OSCILLATORS

Single qubit has been modeled as a system of two coupled limit cycle oscillators, containing random components of their dynamics. Namely, let the initial dynamical equations for limit cycle oscillators, that will be used in qubit model construction, be written as

$$\dot{u}_1 = [\rho_{1,2}^2 + i\,\omega_{1,2} - |u|^2]u$$

where $u_{1,2} = x_{1,2} + iy_{1,2}$ is complex-valued dynamical variables, ρ_1, ρ_2 are radii of circular limit cycles and ω_1, ω_2 are own oscillator frequencies of two identical oscillators [11-13]. Consider further randomly perturbed oscillators of the type, that is, oscillators with limit cycle radii $\tilde{\rho}_1$, $\tilde{\rho}_2$ and own frequencies $\tilde{\omega}_1$, $\tilde{\omega}_2$ chosen as

$$\tilde{\rho}_{1,2} = \rho_{1,2} + \xi_{1,2}(t), \quad \tilde{\omega}_{1,2} = \omega_{1,2} + \eta_{1,2}(t), \tag{11}$$

where $\tilde{\xi}_{1,2}(t)$ and $\tilde{\eta}_{1,2}(t)$ are stationary random functions with zero means. Then the system of ODE governing internal dynamics of two coupled stochastic oscillators can be written as

$$\dot{u}_1 = [\tilde{\rho}_1^2 + i\,\tilde{\omega}_1 - |u_1|^2]u_1 + \kappa(u_2 - u_1),$$
$$\dot{u}_2 = [\tilde{\rho}_2^2 + i\,\tilde{\omega}_2 - |u_2|^2]u_2 - \kappa(u_2 - u_1), \tag{12}$$

where

$$u_1 = x_1 + iy_1, \quad u_2 = x_2 + iy_2, \tag{13}$$

and $\kappa = |\kappa| e^{i\delta}$ is the strength of oscillator connection The variable $U = u_1 + u_2$, defining oscillation superposition of two oscillator - components, will represent the main interest in qubit behavior. So it is convenient to rewrite system (12) for variables $v_1 = 0.5(u_1 + u_2)$, $v_2 = 0.5(u_1 - u_2)$:

$$\dot{v}_1 = 0.5\{[\tilde{\rho}_1^2 + i\,\tilde{\omega}_1 - |v_1 + v_2|^2](v_1 + v_2)$$
$$+ [\tilde{\rho}_1^2 + i\,\tilde{\omega}_1 - |v_1 - v_2|^2](v_1 - v_2)\}$$
$$\dot{v}_2 = 0.5\{[\tilde{\rho}_2^2 + i\,\tilde{\omega}_2 - |v_1 + v_2|^2](v_1 + v_2)$$
$$-[\tilde{\rho}_1^2 + i\,\tilde{\omega}_1 - |v_1 - v_2|^2](v_1 - v_2)\} - 4\kappa v_2, \tag{14}$$

where

$$v_1 = x + iy, \quad v_2 = z + iu. \tag{15}$$

Four-dimensional dynamical system (14) has been constructed in such a manner, that the projection of its trajectory onto (x, y)-plane imitates the behavior of electrical field vector $\vec{E}(t)$ of light beam, that is now presented as the superposition of two beams in the states of right and left circular polarization. The polarization of the summary beam depends on superposition type of beam components. In the case of coherent superposition of two oppositely circularly polarized light beams the summary beam will be in some state of full polarization, whereas in the case of non-coherent superposition it will be in the unpolarized state. Three typical examples of $\vec{E}(t)$ behavior of summary beam in the case of coherent superposition of beam-components are shown in fig. 3 – 5. The electric field of circularly polarized light beam, depicted in fig. 3, is obtained in the case of zero light intensity of the second beam. It corresponds to pure qubit state $|1\rangle$ (photon ensemble of fully circular polarized photons). The electric field of linearly polarized summary light beam, depicted in fig. 4, is obtained in the case of coherent superposition of two oppositely circularly polarized light beams at phase difference $\delta = 0$ for electric field vectors $\vec{E}^{1,2}(t)$. The general case of elliptic polarization, obtained in the case of coherent beams superposition at $\delta \neq 0$, is shown in fig. 5. At last, the electrical field of unpolarized light beam, obtained in the case of non-coherent superposition of two oppositely circularly polarized beams, is presented in fig. 6.

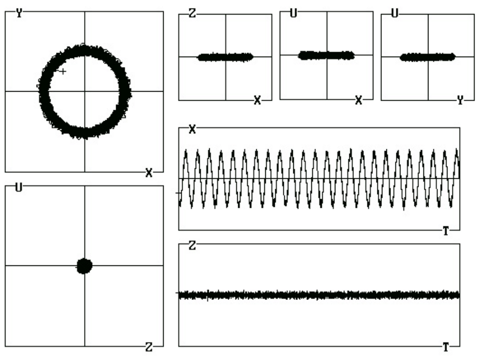

Fig. 3. Pure qubit state $|1\rangle$
(Ensemble of circular polarized photons)

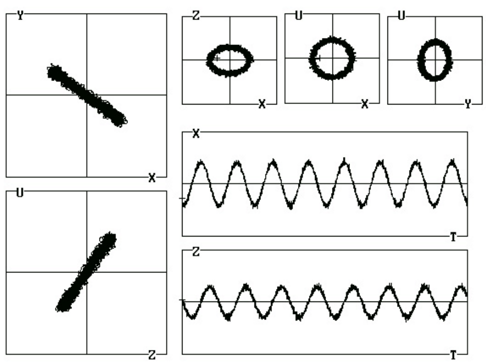

Fig. 4. Pure qubit state $|1\rangle + |0\rangle$
(Ensemble of linearly polarized photons)

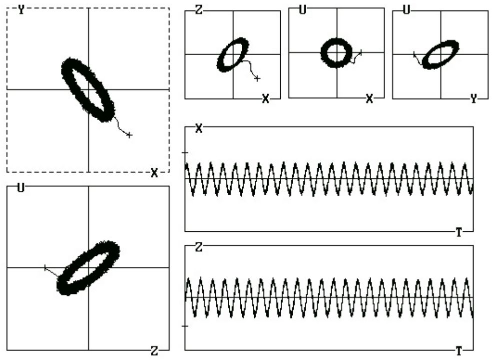

Fig. 5.Pure qubit state $\alpha|1\rangle + \beta|0\rangle$
(Ensemble of elliptically polarized photons)

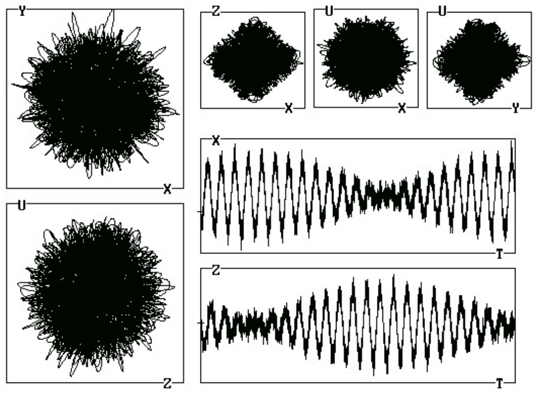

Fig. 6. Mixed qubit state , corresponding to $\hat{\rho} = 0.5 \cdot diag[1,1]$
(Ensemble of unpolarized photons)

So, the designed oscillatory model of qubit correctly simulates light electric field behavior both in typical states of full light polarization (that correspond to pure quantum mechanical states of photon beam) and in the case of unpolarized light (that corresponds to mixed quantum mechanical state of photon beam).

IV. CLUSTER OF ENTANGLED QUBITS. ONE-QUBIT GATES

One-way computation schemes are based on cluster entanglement degree control via a sequence of one-qubit gates acting on single cluster qubits. The operation of arbitrary qubit rotation on the Bloch sphere has been constructed in [7]. We clarified the possibility of classical qubit interpretation as polarized light beam and designed qubit model as stochastic oscillator, imitating the behavior of electric field of electromagnetic wave. Further it seems to be natural to model a strongly entangled qubit cluster as synchronized network of coupled oscillator qubits subjected to a sequence of external actions, transforming single qubit states and, possibly, network connections. Since the strongly entangled cluster can be viewed as fully polarized light beam, one-qubit gates can be naturally constructed as models of optical devices modifying polarization of beam light. So, let us write the system of equations governing dynamics of entangled cluster of *N*-qubits (network of *N* coupled oscillators) of as [11-13]

$$\dot{\vec{V}}^j = \vec{f}(\vec{V}^j;\, \alpha^j) + \sum_{k=1}^{N} \hat{W}^{jk} \cdot (\vec{V}^k - \vec{V}^j) + \vec{F}(\vec{V}^j;\, \beta^j),$$

$$j = 1, \dots N, \qquad (15)$$

where $\vec{V}^j = (x^j, y^j, z^j, u^j)^T$ is four–component variable, specifying qubit model state, $\alpha^j = \{\tilde{\rho}_1^j, \tilde{\rho}_2^j, \tilde{\omega}_1^j, \tilde{\omega}_2^j, \kappa^j\}$ is the collection of internal qubit parameters (see eq. (14)-(15)), $[\hat{W}^{jk}]$ is the matrix connections of oscillatory qubit network (as a rule, it will be the matrix of all-to-all connections depending on network oscillator states), $\vec{F}(\vec{V};\, \beta^j)$ is four-component function, specifying external action on *j*-th qubit (that is, one-qubit gate). In view of optical interpretation of our oscillatory qubits, the one-qubit gates should imitate the actions of typical optical devices capable to modify polarization of light. A polarizer is just one of widely used optical devices that transforms light polarization. It converts a beam of arbitrarily polarized light into beam with well-defined light polarization, for instance, linear polarization. We are able to model the action of linear polarizer in frames of our qubit model. Let θ be the angle between the direction of polarizer plane of polarization and direction of $\vec{e}_x$ -vector, and $\vec{E}$ be electric field vector of incident light beam. Then electric field vector $\vec{E}'$ of transmitted light is $\vec{E}' = \hat{A}\vec{E}$ ($\hat{A}$ is the matrix of the linear polarizer). In frames of our model the one-qubit gate, imitating qubit passing through linear polarizer of some finite thickness d, can be defined by function $\vec{F}(\vec{V};\, \theta, \Delta t)$ that is nonzero only during finite time interval $\Delta t = t_2 - t_1$, $\Delta t \sim d$. The example of analytical expression for such $\vec{F}(\vec{V};\, \theta, \Delta t)$ can be written as

$$\vec{F}(\vec{V};\, \theta, \Delta t) = \begin{bmatrix} \vec{F}_1 \\ \vec{F}_2 \end{bmatrix} = \frac{d}{dt}\{H(t) \cdot \begin{bmatrix} \hat{A}\vec{v}_1 \\ \hat{A}\vec{v}_2 \end{bmatrix}\}, \qquad (16)$$

$$\hat{A} = \begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \cos^2(\theta) \end{pmatrix}, \qquad (17)$$

$$H(t) = 0.5\{th(\gamma(t - t_1)) - th(\gamma(t - t_2))\},\ \gamma \gg 1, \qquad (18)$$

$$\vec{V} = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix},\ \vec{v}_1 = \begin{bmatrix} x \\ y \end{bmatrix},\ \vec{v}_2 = \begin{bmatrix} z \\ u \end{bmatrix}. \qquad (19)$$

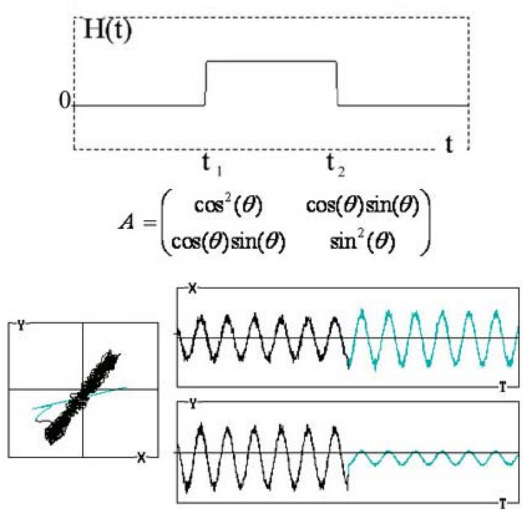

Fig. 7 Single qubit under action of one-qubit gate, imitating passing through linear polarizer (oscillations after transmission are shown by curves of green color)

As one can see from fig. 7, linear polarization of oscillatory qubit has been transformed into another linear polarization after transmission through one-qubit gate, corresponding to linear polarizer action. In the case gate simulates the action of absorptive polarizer which absorbs all the unwanted polarization states besides the one inherent to polarizer. Although there exist also so called beam-splitting polarizers that split the unpolarized light beam into two beams with opposite polarization states, it is just the gate, corresponding to absorptive polarizer will be of the main interest in CQC problems .

Besides one-qubit gate, imitating polarizer action, in a similar manner one-qubit gates, corresponding to phase-shifters (polarization rotators) and optical compensators can be designed as well. Phase-shifters transform a linearly polarized light beam into beam of circularly polarized light via creating of additional phase difference between two components of electric field $\vec{E}$. Matrix $\hat{A}$ of optical compensator in the complex-valued basis $\{\vec{e}^+, \vec{e}^-\}$, $\vec{e}^{\pm} = (1/\sqrt{2})(\vec{e}_x \pm i\vec{e}_y)$ can be written as

$$\hat{A}_c = \begin{pmatrix} \cos(\delta/2) & \pm\sin(\delta/2) \\ \mp\sin(\delta/2) & \cos(\delta/2) \end{pmatrix}. \qquad (20)$$

Thus, different kinds of one-qubit gates, modeling actions of external optical devices on single network qubits, can be easily constructed in the frames of our model. The gates are specified by functions $\vec{F}(\vec{V};\, \beta^j)$, that figure at right-hand sides of dynamical system (15), governing dynamics of oscillatory network. The dynamical system (15) is intended to describe evolution of synchronized oscillatory network under gradual predefined state modifications of network processing units. The oscillator state modifications will be specified by CQC algorithm and expressed in construction of functions $\vec{F}(\vec{V};\ \beta^{j})$. If network connections will be constructed as functions of oscillator states (as it was done in [11-13]) , the network synchronization state will also be capable to gradual modifications.

V. SUMMARY

We suggested an interpretation of one-way quantum computations as state evolution of synchronized network of stochastic oscillators. In the way the following issues are elucidated:

- a beam of quasi-monochromatic polarized light can be used as a computation resource in one-way quantum computation schemes;

- single qubit can be also viewed as a beam polarized light;

- oscillatory model of qubit is designed in the paper as stochastic oscillator formed by two coupled limit cycle oscillators, which oscillation amplitudes and own frequencies are stationary functions of time; the model adequately imitates both pure and mixed states of two-level quantum-mechanical system;

- synchronized network of coupled stochastic oscillators is suggested as a model of cluster of entangled qubits; the number of network oscillators is equal to the number of controllable beam components contained in initial light beam;

- one-qubit gates can be modeled as actions of typical optical devices; an example of the gate, imitating the action of polarizer, is constructed in the paper;

- system of equation, governing dynamics of oscillatory network is written;

- the suggested model can be considered as quantum-inspired neuro-evolutionary model: it allows to formulate the one-way quantum computation scheme as dynamical evolution of controllable artificial feed-forward neural network under gradual variation of both network processing unit states and network interconnection architecture .

The approach could be helpful both for computer study of behavior entangled qubit cluster model and for development of methods of cluster evolution control.

REFERENCES

[1]. P.W.Shor, "Polynomial time algorithms for prime factorization and discrete algorithms on a quantum computer, *SIAM J. Sci. Statist. Comput*. v.26, p. 1484, 1997.

[2] A.S.Holevo, "Probabilistic and statistical aspects of quantum theory". Nauka, Moscow, 1980; North Holland Translation - 1982

[3] A.S.Holevo, " Quantum probability and quantum statistics", VINITI, v.83 Moscow, 1991 (in Rusian)

[4]. M.A.Nielsen and I.L.Chuang, *Quantum computation and quantum information*, Cambridge Univ. Press, 2000.

[5]. R.Raussendorf and H.J.Breigel, "A one way quantum computer", *Phys. Rev. Lett.* V.86, p.5188, 2001.

[6]. R.Raussendorf and H.J.Breigel, "Computational model underlying the one-way quantum computer", *Quant. Inf. Comp.*, v.6, p. 443, 2002; *quant-ph/* 0108063 (2001).

[7]. R.Raussendorf and H.J.Breigel, Computational model underlying the one-way quantum computer: concepts and summary", in: Th.Beth ans G. Leuch, *Quantum Information processing*, Wiley-VCH, 2003.

[8]. L.K.Grover, "A fast quantum mechanical algorithm for database search", *Proc. 28 Annual ACM Symp. on Theory of Computing*, p.22, 1996.

[9]. I.L.Chuang, L.M.K.Vandersypen, X.Zhou, D.W.Leung, S.Lloyd, "Experimental realization of a quantum algorithm", *Nature* v. 393, p.143, 1998.

[10]. L.M.K. Vandersypen, I.L.Chuang, "NMR technique for quantum control and computation", *arXiv: quant-ph/*0404064 v.2, 2004.

[11] M. G. Kuzmina, E. A. Manykin and I. I. Surina "Oscillatory network with self-organized dynamical connections for synchronization-based visual image segmentation", *BioSystems*, vol. 76, pp. 43-53, 2004.

[12] E.S Grichuk, M. G. Kuzmina, E. A. Manykin , Oscillatory Network for Synchronization-based Adaptive Image Segmentation, *Proc. of WCCI 2006*, Vancouver, BC, Canada; WCCI 2006 CD, IEEE Catalog Number: 06CH37726D, ISSN: 0-7803-9489-5, 347-352 (2006) ; *Proc. of Int. Joint Conf on Neural Networks 2006*, p.4529, 2006.

[13] Grichuk E.S., Kuzmina M.G., Manykin E.A., Adaptive image processing via synchronization in self-organizing oscillatory network, *Proceedings of European Computing Conference (ECC 2007*), Athens, Greece; *Lecture Notes in Electrical Engineering*, (Eds. N.Mastorakis, V.Mladenov, V.T.Kontargyri), Berlin: Springer, , v.1, p.103, 2009.