Oscillatory Network Model of the Brain Visual Cortex with Controlled Synchronization

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Abstract

Oscillatory network for modelling of synchronization-based functioning of the brain visual cortex is presented. Single network oscillator, imitating the behavior of simple cells of the visual cortex, demonstrates stimulus-dependent intrinsic dynamics — stable oscillations or quick relaxation. Three-dimensional spatial architecture of the network simulates the columnar structure of the visual cortex. Nonlinear nonlocal dynamical interaction of oscillators depends on instantaneous oscillator activities and orientations of receptive fields. Two-dimensional averaged network of idealized oscillator-columns is extracted.

The model demonstrates controlled synchronization of image-dependent clusters of oscillatory network and self-controlled suppression of noisy background.

1 Introduction

The brain visual cortex (VC) realizes solutions of a variety of essentially different tasks of visual processing. Since the experimantal discovery of synchronous oscillations in the VC of cat and monkey [1,2] (and later in other brain areas) the viewpoint was adduced that syncronization-based functioning is inherent to VC in problems of visual processing.

Series of attempts was enterprised to elucidate the role of cortical oscillations and synchronization in visual image processing. A number of network models with various types of oscillators as processing units was designed in the 90th and studied in the context of visual image segmentation problems [3-7].

Oscillatory network of columnar architecture imitating functioning of the primary VC in contour integration task was designed in [8]. It was shown that synchronization of network ensembles is facilitated for smooth, long and closed contours.

The performance of the presented model is based on syncronization of network ensembles controlled by visual image characteristics. We concentrate so far on images of simple structure supposing that no preprocessing (filtering) caused by multi-scale image structure is necessary. Following [8] we designed the network of columnar architecture. However, several features inherent in our model differ it essentially from the model [8]. First - the dynamics of single oscillator is tuned by two visual image characteristics - local contrast and elementary bar orientation. Second - network connections are defined directly in terms of oscillator interaction rather than in terms of excitatory and inhibitory connections of neurons forming oscillators. Third - the designed dynamical connections are of threshold character and strongly depend on RF orientations. These properties of our model just provide the promising network features — synchronization of network clusters, encoded by incoming visual image, and self-controlled suppression of noisy background.

2 Columnar Oscillatory Network Model

Modelling VC as a network of coupled oscillators we imply, following [8], that single oscillator is formed by a pair of interconnected excitatory and inhibitory neurons.

The oscillatory network model of the VC is designed as the network of columnar architecture consisting of N^2 columns of K oscillators each ($N^2 \cdot K$ is the total number of oscillators). The bases of the columns are located at the nodes of 2D square lattice G_{N^2}, whereas oscillators of each column are located at the nodes of 1D lattice L^K oriented normally with respect to the plane of G_{N^2}. So the oscillators of the whole network are located at the nodes of 3D lattice $G_{N^2} \times L^K$.The location of a single oscillator is specified by a radius-vector $\mathbf{r}^k_{lm} = (x^k_{jm}, y^k_{jm}, z^k_{jm})$. The state of the network is specified by $(N \times N \times K)$-matrix of oscillator states $[u^k_{jm}]$. For each oscillator the orientation of its RF is specified by 2D unit vector $\mathbf{n}^k_{jm}$, which is an important internal parameter. In accordance with [9] the orientatioins $\mathbf{n}^k_{jm}$ are assumed deterministically uniformly distributed over the columns. The retina is modelled by 2D square lattice similar to G_{N^2}. So, a continuous visual image arising in real retina is represented by its discretization in the retina lattice, that is, by a collection of pairs $(I_{jm}, \mathbf{s}_{jm}), j = 1, \ldots, N, m = 1, \ldots, N$, where I_j is local contrast and $\mathbf{s}_{jm}$ — local orientation of image elementary bar. Suitable type of dynamics imitating stimulus-dependent response of simple cells in the VC is delivered by oscillator with two degrees of freedom. Defining oscillator state by two-component real-valued vector function $\mathbf{u} = (u_1, u_2)^\top$, the system of two coupled differential equations for u_1, u_2 can be written in the form of single equation for complex-valued function $u = u_1 + i \cdot u_2$:

$$\dot{u} = (\rho^2 + i\omega - |u - c|^2)(u - c) + \lambda_0(1 - g(I - h_0) + q(\mathbf{s}, \mathbf{n})). \qquad (1)$$

Here ρ, c, ω are constants defining asymptotic parameters of the limit cycle of dynamical equation (1): at $\lambda_0 = 0$ the limit cycle is the circle of radius ρ with center location at the point $c = |c|e^{i\alpha}$ in the complex u-plane, ω is the cycle frequency. The constant λ_0 is a complex (tuning) constant. Properly constructed functions g and q dependent on visual image characteristics $(I, \mathbf{s})$ provide controlling of bifurcation parameter $\lambda = \lambda_0(1 - g + q)$: the Hopf bifurcation occurs at some $\lambda = \lambda_*, \quad \lambda_* \in (0, 1)$ (limit cycle is converted into stable focus located in the vicinity of the origin).

Constructing g and q we exploited threshold character of dependence of oscillator response on visual image contrast I and sharp peak-shaped dependence on $|\mathbf{s} - \mathbf{n}|$. In eq.(1) we used the functions $g(I - h_0) = 1/(1 + e^{-2\nu(I - h_0)})$, and $\Gamma(|\phi|) = 2e^{-\sigma|\phi|}/(1 + e^{-2\sigma|\phi|})$, where $\phi = \psi - \beta$, the angles ψ and β define the orientations of vectors $\mathbf{n}$ and $\mathbf{s}$, respectively ($\mathbf{s} = (\cos\beta, \sin\beta)$, $\mathbf{n} = (\cos\psi, \sin\psi)$).

The dynamical system governing the dynamics of the oscillatory network can be written as

$$\dot{u}^k_{jm} = f(u^k_{jm}, \lambda(I_{jm}, \mathbf{s}_{jm}, \mathbf{n}^k_{jm})) + S^k_{jm};$$

$$j, m = 1, \ldots, N, \quad k = 1, \ldots, K. \qquad (2)$$

Here $f(u, \lambda) = (\rho^2 + i\omega - |u - c|^2)(u - c) + \lambda, \quad \lambda = \lambda_0(1 - g(I_{jm} - h_0) + q(\mathbf{s}_{jm}, \mathbf{n}^k_{jm}))$ and the term S^k_{jm} specifies interaction between oscillators in the network. It can be written as

$$S^k_{jm} = \sum_{j', m', k'} W^{kk'}_{jj'mm'}(u^k_{jm}, u^{k'}_{j'm'})(u^{k'}_{j'm'} - u^k_{jm}), \qquad (3)$$

where the elements of matrix of connections W are represented in the factorized form:

$$W^{kk'}_{jj'mm'} =$$

$$P^{kk'}_{jj'mm'}(u, u')Q^{kk'}_{jj'mm'}(\mathbf{n}, \mathbf{n}')D^{kk'}_{jj'mm'}(|\mathbf{r} - \mathbf{r}'|), \qquad (4)$$

$\mathbf{r}$ and $\mathbf{r}'$ are radius-vectors, defining spatial locations of oscillators (j, m, k) and (j', m', k').

In (4) factor $P^{kk'}_{jj'mm'}$ defines the dependence of connections on the product of oscillator states (of threshold character) , factor $Q^{kk'}_{jj'mm'}$ — the dependence on RF orientations and $D^{kk'}_{jj'mm'}$ — on spatial distance in the network.

3 2D Averaged Oscillatory Network.

It is helpful to introduce a network of simplified architecture defined in the lattice G_{N^2} and closely related to the columnar one. This averaged network, consisting of idealized oscillator-columns, can be derived from the columnar network as a result of inter-column averaging and special limit analogous to well-known thermodynamical limit in statistical physics. The state of the averaged network is defined by $N \times N$ matrix $[u_{jm}]$. The RF orientation $\mathbf{n}_{jm}$ of its single oscillator coincides with the stimulus bar orientation $\mathbf{s}_{jm}$. The internal dynamics of the oscillator is governed by eq.(1) with $q(\mathbf{s}, \mathbf{n}) \equiv \mathbf{1}$, and the elements of network matrix of connections are: $\bar{W}_{jj'mm'} = \bar{W}_{jj'mm'}(u, u'; \mathbf{s}_{jm}, \mathbf{s}_{j'm'})$.
Thus, dynamical equations of the reduced network can be written as

$$\dot{u}_{jm} = \bar{f}(u_{jm}, \lambda) + \sum_{j',m'=1}^{N} \bar{W}_{jmj'm'}(u_{j'm'} - u_{jm}),$$

$$j = 1, \ldots, N, \quad m = 1, \ldots, N. \tag{5}$$

In the case of local dynamical connections the nonlinear diffusion equation (reaction-diffusion equation) governing the dynamics of oscillatory medium — continual analogy of the reduced oscillatory network — can be derived. Some results of qualitative analysis of reaction-diffusion equation for oscillatory media with kinetics (1) at $\lambda_0 = 0$ were obtained in [10].

4 Computer Experiments

The initial series of computer experiments was carried out with 2D averaged oscillatory network. The first step was to design properly tuned dynamics of single oscillator. Sharp oscillator response on visual stimulus characteristics was achieved, Figs. 1-2.
The second step consisted in testing the abilities of interaction (4). It was found out that the interaction provides the desirable self-controlled coupling. Namely, it becomes weak in the cases:
a) one of the oscillator activities $|u_1|, |u_2|$ is close to background;
b) the RF orientations are not close to each other;
c) the distance between oscillator locations exceeds the radius of interaction.
The example of two-oscillator dynamics is shown in Fig.3. At the absence of coupling the first oscillator has the limit cycle shown in (u_1, u_2)-plane, whereas the second one has the stable node denoted by P in (v_1, v_2)-plane. When the coupling is switched on, synchronization occurs if coupling strength w exceeds the threshold value w_*. The limit cycle of small size arises in (v_1, v_2)-plane, and relaxational dynamics is changed into oscillations.
The last series of experiments concerned 1D averaged network. Synchronization of ensembles encoded by 1D contour of sufficient contrast and slowly varying bar orientations was observed.

Conclusive Remarks

The columnar oscillatory network model is proposed. The main feature of the model is the combination of oscillator dynamics parametrically tunable by visual image characteristics and nonlocal dynamical oscillator interaction dependent on instantaneous oscillator states and orientations receptive fields.
Visual image contour detection has been observed in the initial series of computer experiments with the averaged network model.

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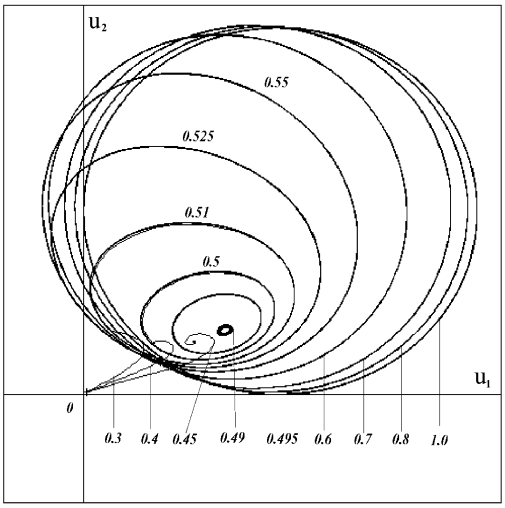

Fig. 1.

The dependence of oscillator limit cycle on visual image contrast *I*. The limit cycle bifurcates into focus at $I < I^*$.

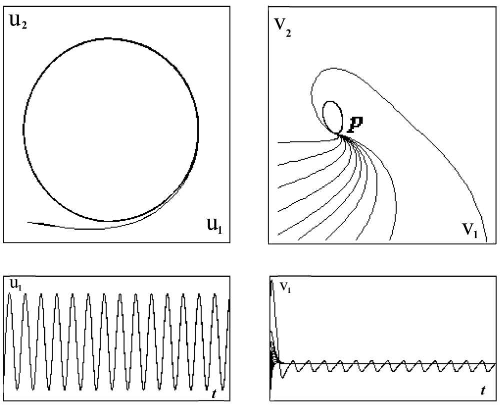

Fig. 3.

Two oscillators coupled via connection (4). Synchronization in the case of essentially different I_1 and I_2.

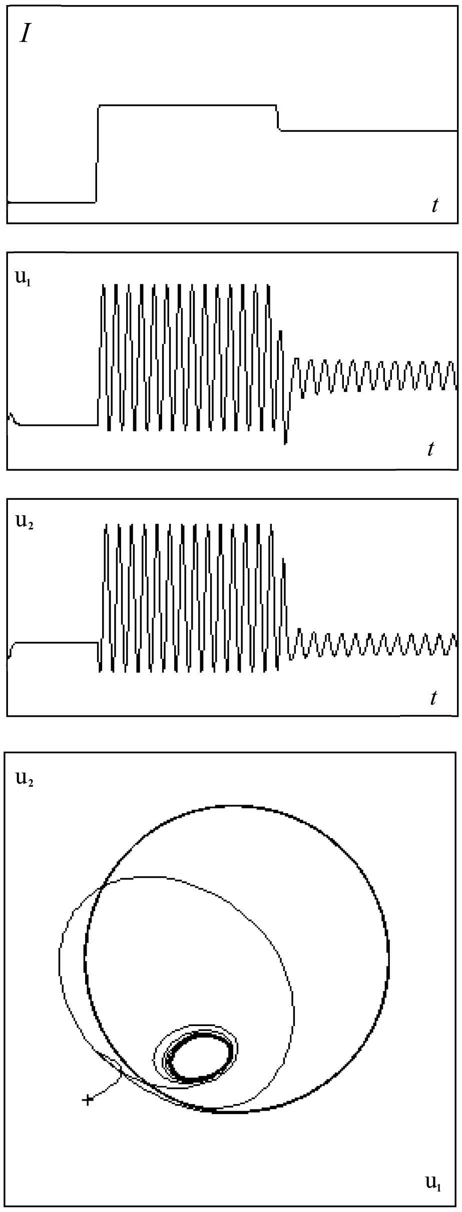

Fig. 2.

The response of network oscillator to "switching on" of visual image and sharp variation of its contrast. (Time dependences and phase portrait.)