Methods of satellite formation flying control

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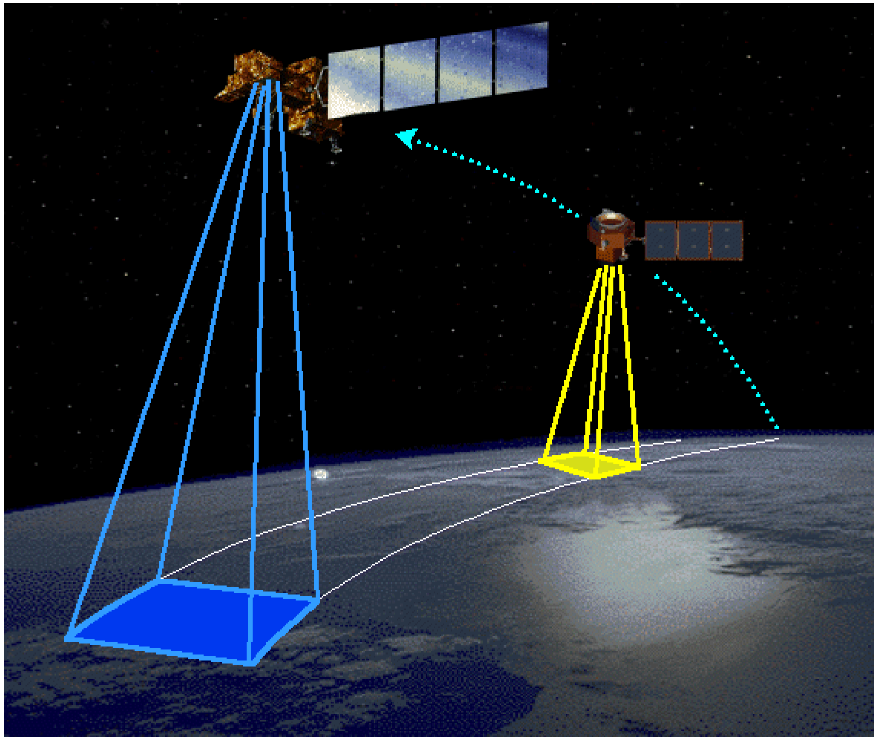
Outline

- What is formation flying
- Model of motion
  - Relative motion equations
  - Clohessy-Wiltshire equations
- Methods of control

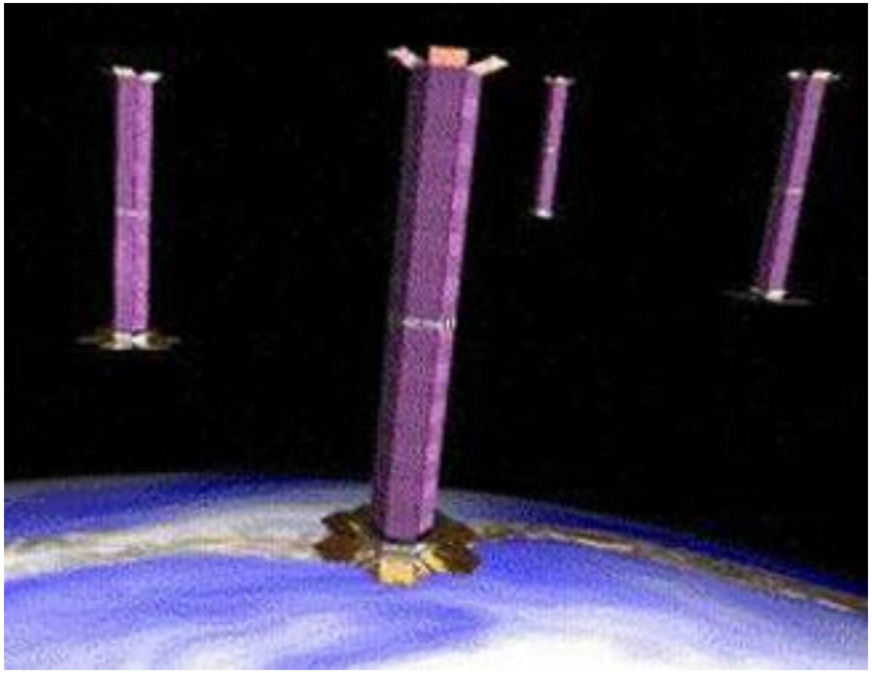
What is Formation Flying

Formation flying is multiple satellites orbiting in close proximity in a cooperative manner

Examples

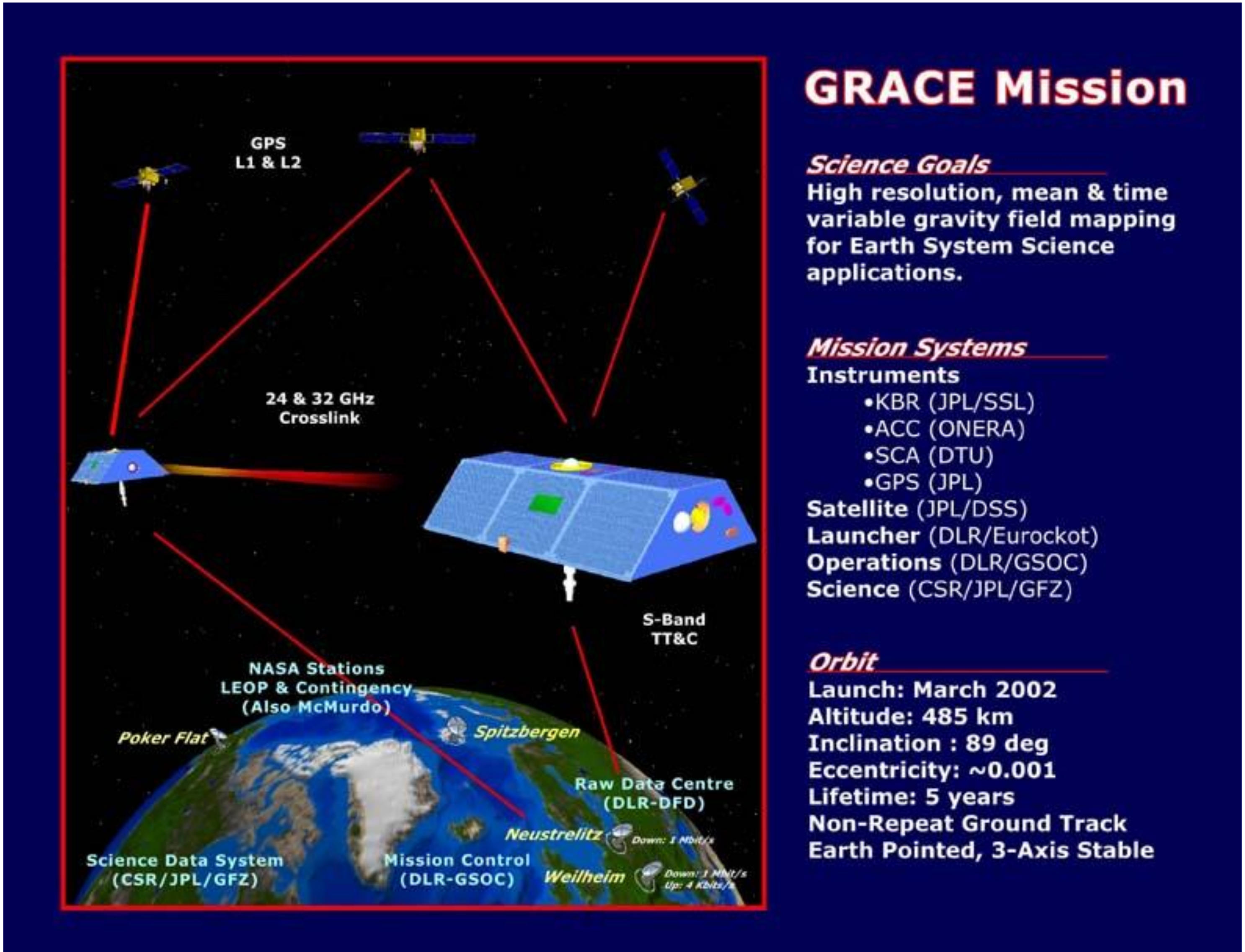


Landsat-7 being trailed by EO-1 covering the same area at different times



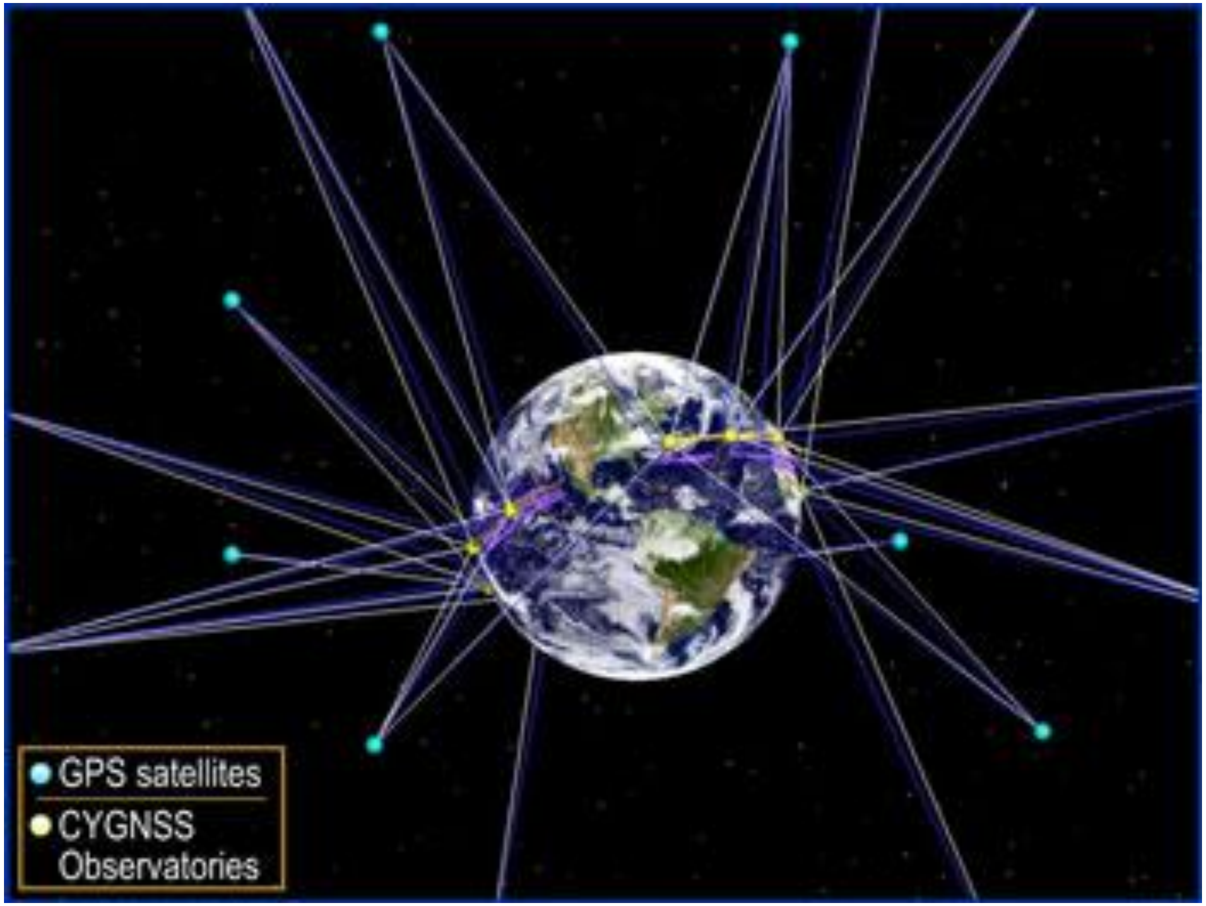
TechSat-21 cluster formation

Examples


GPS
L1 & L2
24 & 32 GHz
Crosslink
S-Band
TT&C
NASA Stations
LEOP & Contingency
(Also McMurdo)
Poker Flat
Spitzbergen
Raw Data Centre
(DLR-DFD)
Neustrelitz
Down: 1 Mbit/s
Science Data System
(CSR/JPL/GFZ)
Mission Control
(DLR-GSOC)
Weilheim
Down: 1 Mbit/s
Up: 4 Kbits/s
GRACE Mission
Science Goals
High resolution, mean & time variable gravity field mapping for Earth System Science applications.
Mission Systems
Instruments
•KBR (JPL/SSL)
•ACC (ONERA)
•SCA (DTU)
•GPS (JPL)
Satellite (JPL/DSS)
Launcher (DLR/Eurockot)
Operations (DLR/GSOC)
Science (CSR/JPL/GFZ)
Orbit
Launch: March 2002
Altitude: 485 km
Inclination : 89 deg
Eccentricity: ~0.001
Lifetime: 5 years
Non-Repeat Ground Track
Earth Pointed, 3-Axis Stable

Examples

NASA's Weather Prediction Project
The Cyclone Global Navigation Satellite System (CYGNSS)



Problem

- Precise relative position is important
- Precision control often is not important
- Minimum fuel control is essential

So we need

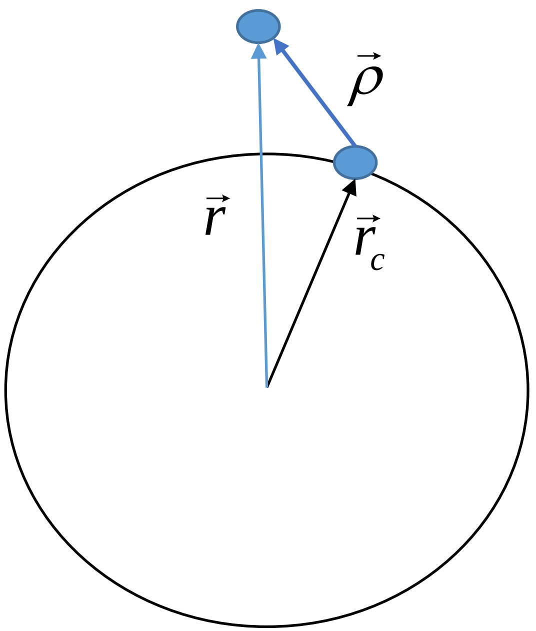
- Accurate mathematical model of motion
- Methods of control with low fuel consumption

Model of Motion

- Two satellites are close to each other
- Study relative motion

Newton's gravity: $\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$

ω is an angular velocity of first satellite



Relative motion is

$$\ddot{\vec{\rho}} + 2\vec{\omega}\times\dot{\vec{\rho}} + \dot{\vec{\omega}}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \ddot{\vec{r}}_c = -\left(\frac{\mu}{r^3}\vec{r} - \frac{\mu}{r_c^3}\vec{r}_c\right)$$

Relative Motion Equations

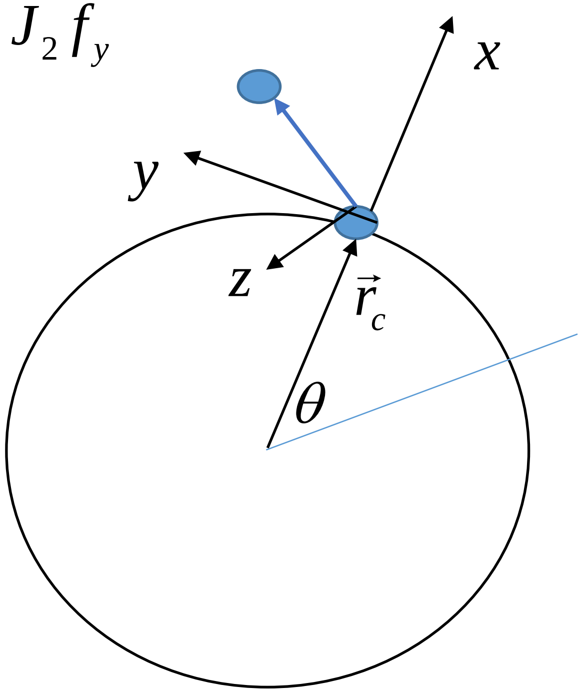
$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r_c^2} + J_2 f_x$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + J_2 f_y$$

$$\ddot{z} = -\frac{\mu z}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + J_2 f_z$$

$$\ddot{r}_c = r_c\dot{\theta}^2 - \frac{\mu}{r_c^2}[1 + J_2 f_r]$$

$$\ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c}[1 + J_2 f_\theta]$$



Relative Motion Equations

Need to simplify the equations

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r_c^2} + J_2 f_x$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + J_2 f_y$$

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Relative Motion Equations

Spherical Earth

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r_c^2} + \cancel{J_2 f_x}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \cancel{J_2 f_y}$$

$$\ddot{z} = -\frac{\mu z}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \cancel{J_2 f_z}$$

$$\ddot{r}_c = r_c\dot{\theta}^2 - \frac{\mu}{r_c^2}[1 + \cancel{J_2 f_r}]$$

$$\ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c}[1 + \cancel{J_2 f_\theta}]$$

Relative Motion Equations

Circular reference orbit

$$\ddot{x} - 2\dot{\theta}\dot{y} - \cancel{\ddot{\theta} y} - \dot{\theta}^2 x = -\frac{\mu (r_c + x)}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r_c^2}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \cancel{\ddot{\theta} x} - \dot{\theta}^2 y = -\frac{\mu y}{[(r_c + x)^2 + y^2 + z^2]^{3/2}}$$

$$\ddot{z} = -\frac{\mu z}{[(r_c + x)^2 + y^2 + z^2]^{3/2}}$$

$$\ddot{r}_c = r_c \dot{\theta}^2 - \frac{\mu}{r_c^2} = 0$$

$$\ddot{\theta} = -\frac{2\dot{r}_c \dot{\theta}}{r_c} = 0$$

Relative Motion Equations

Linearizing

$$\ddot{x} - 2\dot{\theta}\dot{y} - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r_c^2} = 2\dot{\theta}^2 x$$

$$\ddot{y} + 2\dot{\theta}\dot{x} - \dot{\theta}^2 y = -\frac{\mu y}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} = -\dot{\theta}^2 y$$

$$\ddot{z} = -\frac{\mu z}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} = -\dot{\theta}^2 z$$

$$r_c = \text{const}$$

$$\dot{\theta} = \sqrt{\frac{\mu}{r_c^3}} = n$$

Clohessy-Wiltshire Equations

Assumptions:

- Spherical Earth
- Circular reference orbit
- Linearized equations

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0$$

$$\ddot{y} + 2n\dot{x} = 0$$

$$\ddot{z} + n^2 z = 0$$

$$x = 2(2x_0 + \dot{y}_0 / n) - (3x_0 + 2\dot{y}_0 / n)\cos nt + (\dot{x}_0 / n)\sin nt$$

$$y = (y_0 - 2\dot{x}_0 / n) - 3(2x_0 + \dot{y}_0 / n)nt + (2\dot{x}_0 / n)\cos nt + 2(3x_0 + 2\dot{y}_0 / n)\sin nt$$

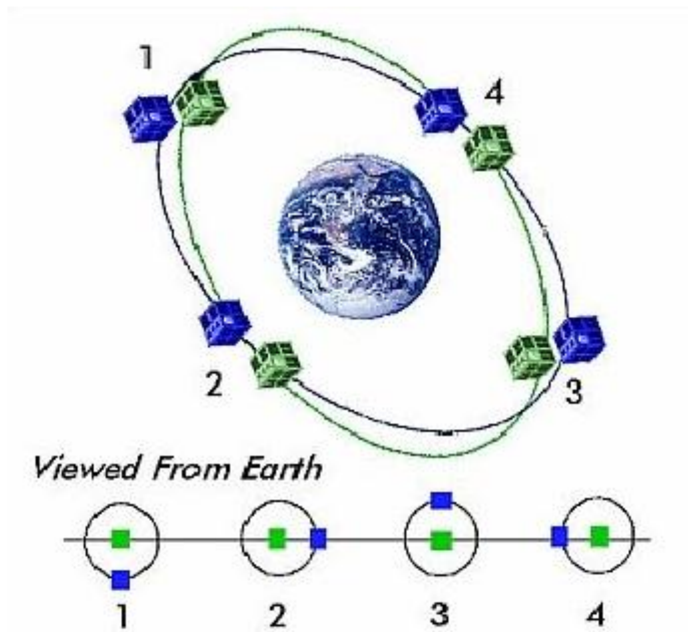
$$z = z_0 \cos nt + (\dot{z}_0 / n)\sin nt$$

Clohessy-Wiltshire Equations

Periodic solutions

- Leader-Follower, $x = z = 0, \quad y = \text{const}$
- 2-1 ellipse, $x = A\sin(nt+\alpha), y = 2A\cos(nt+\alpha), \text{z=Bsin}(nt+\beta)$
- Circle, $\alpha = \beta, B = \sqrt{3}A, \quad x^2 + y^2 + z^2 = 4A^2$
- Projected Circular Orbit,

$$A = B/2, \alpha = \beta, \ y^2 + z^2 = B^2$$



Clohessy-Wiltshire Equations

$$\ddot{x} - 2n\dot{y} - 3n^2 x = u_x + O(\rho / R) + O(e) + O(J_2)$$

$$\ddot{y} + 2n\dot{x} = u_y + O(\rho / R) + O(e) + O(J_2)$$

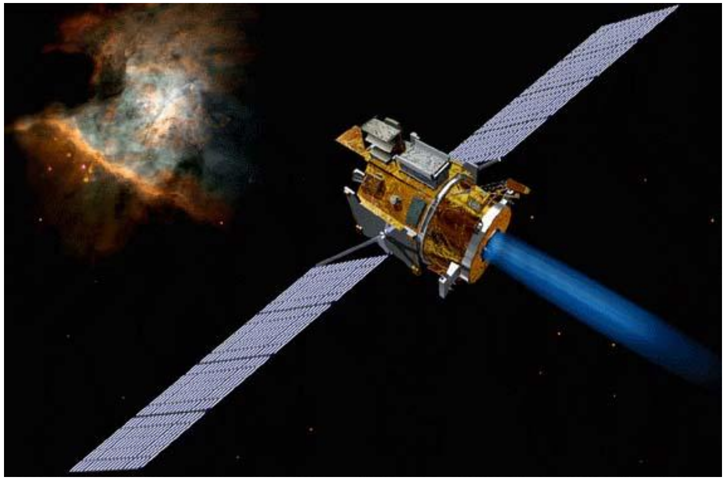
$$\ddot{z} + n^2 z = u_z + O(\rho / R) + O(e) + O(J_2)$$

(u_x, u_y, u_z) are control forces

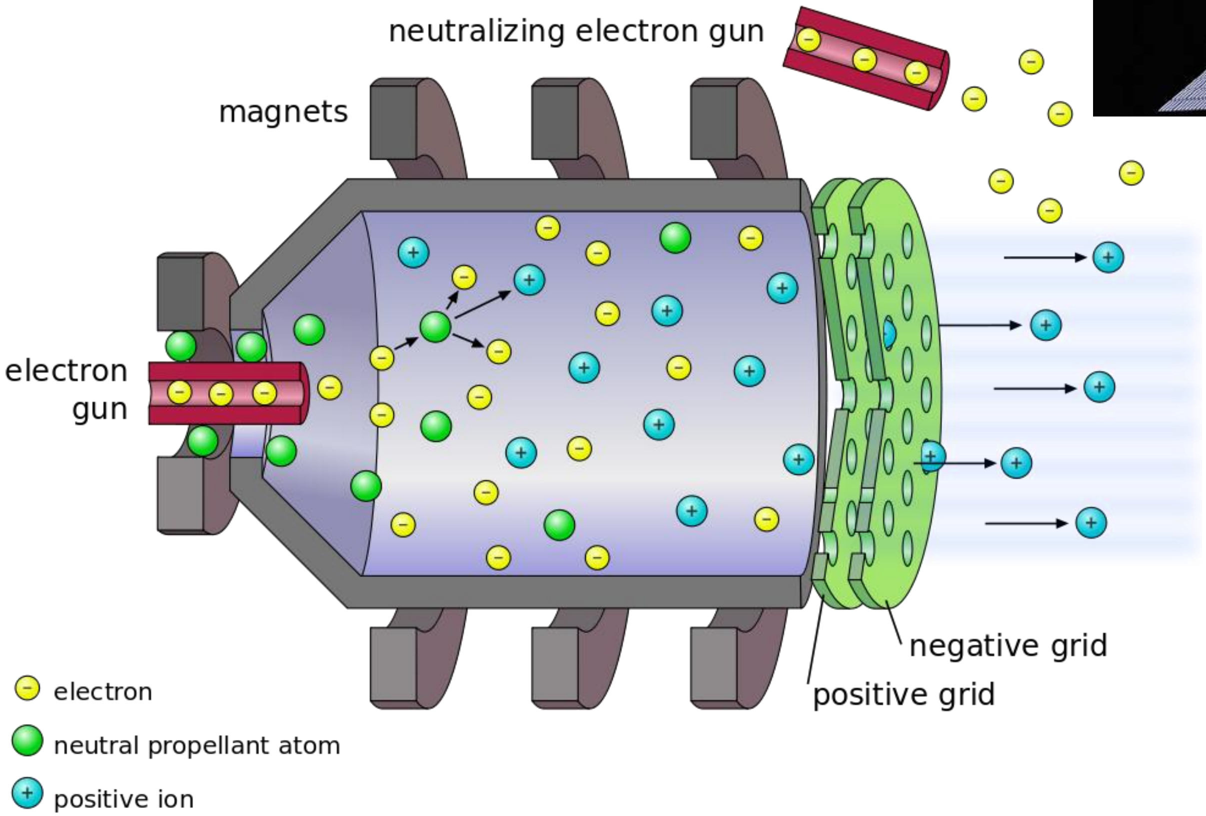
Neglected terms in general are of the same order

System can usually be described by linear equations

Methods of Control

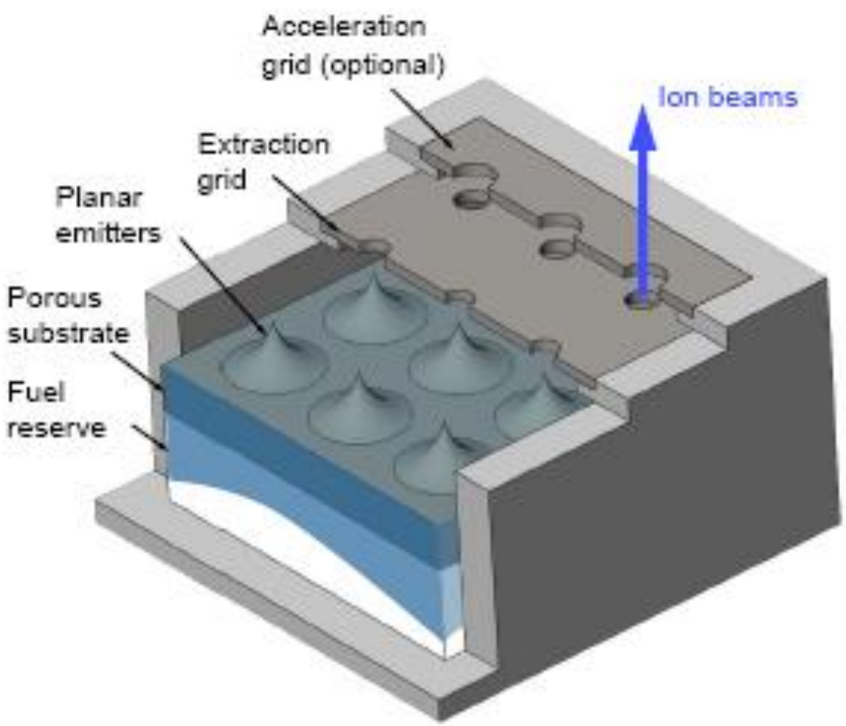


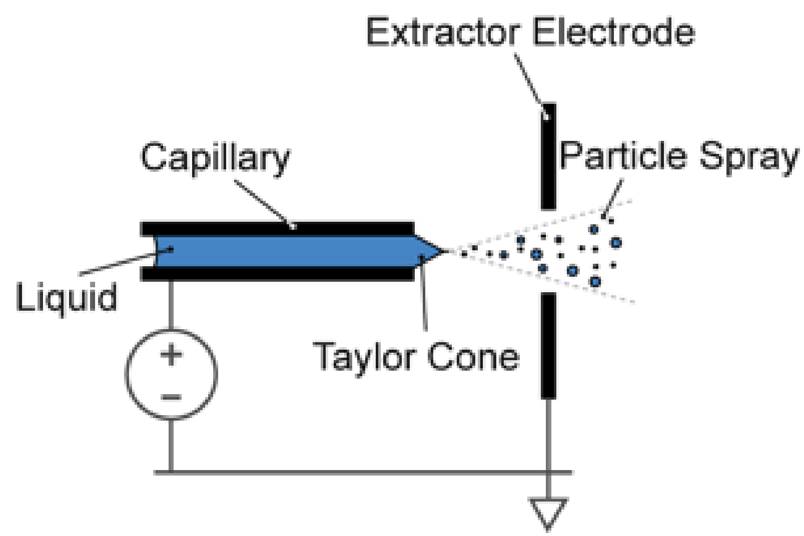
Ion thruster



Methods of Control

Colloid thruster

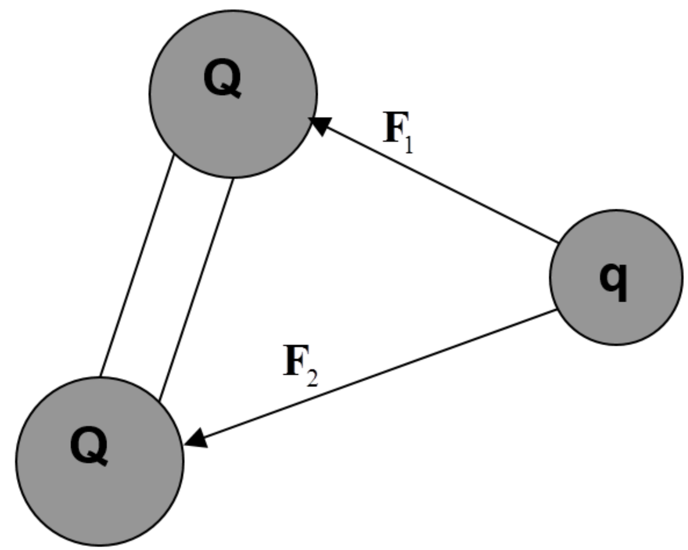




Methods of Control

Electromagnetic force

- Every satellite in a group can accumulate electric charge
- Electrostatic fields of satellites interact (10 – 100 m.)
- Attitude control due to difference between forces
- Charged satellite interact with geomagnetic field

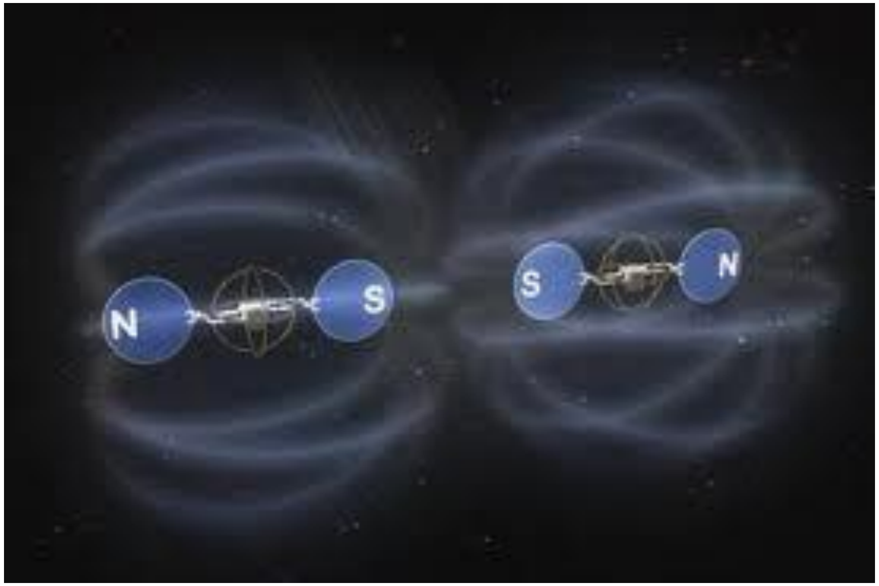


1979 г. --- SCATHA (Spacecraft Charging At High Altitudes)
2001 г. --- NASA Institute for Advanced Concepts

Methods of Control

Magnetorquers

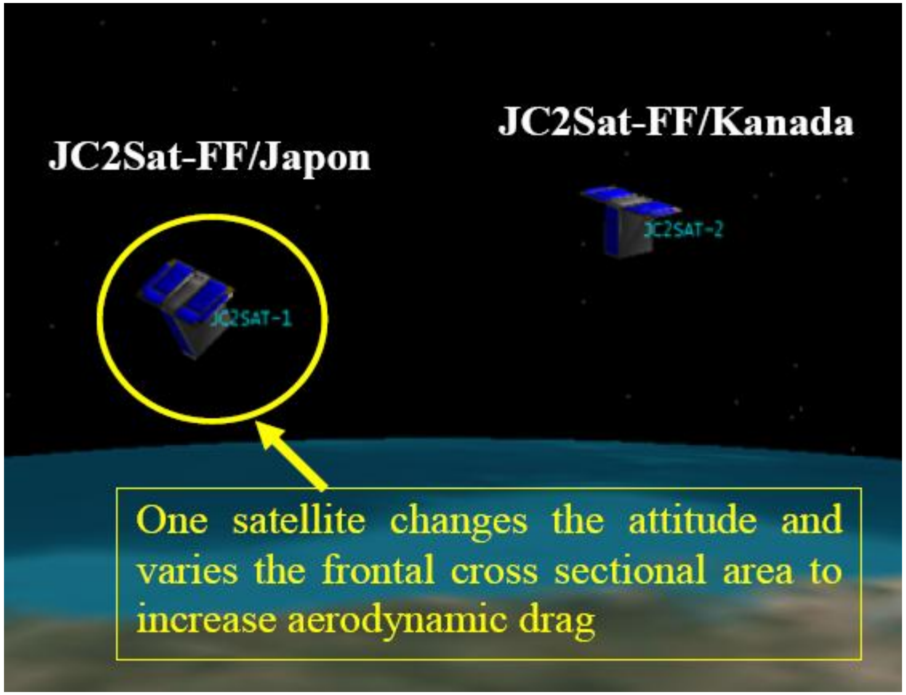
- Magnetic torquer is a system for attitude control built from electromagnetic coils
- Develop magnetic field which interfaces with an ambient magnetic field: the other satellite’s or geomagnetic
- Electric current is safer than charge
- Possible future superconductors technologies



Methods of Control

Aerodynamic drag effect

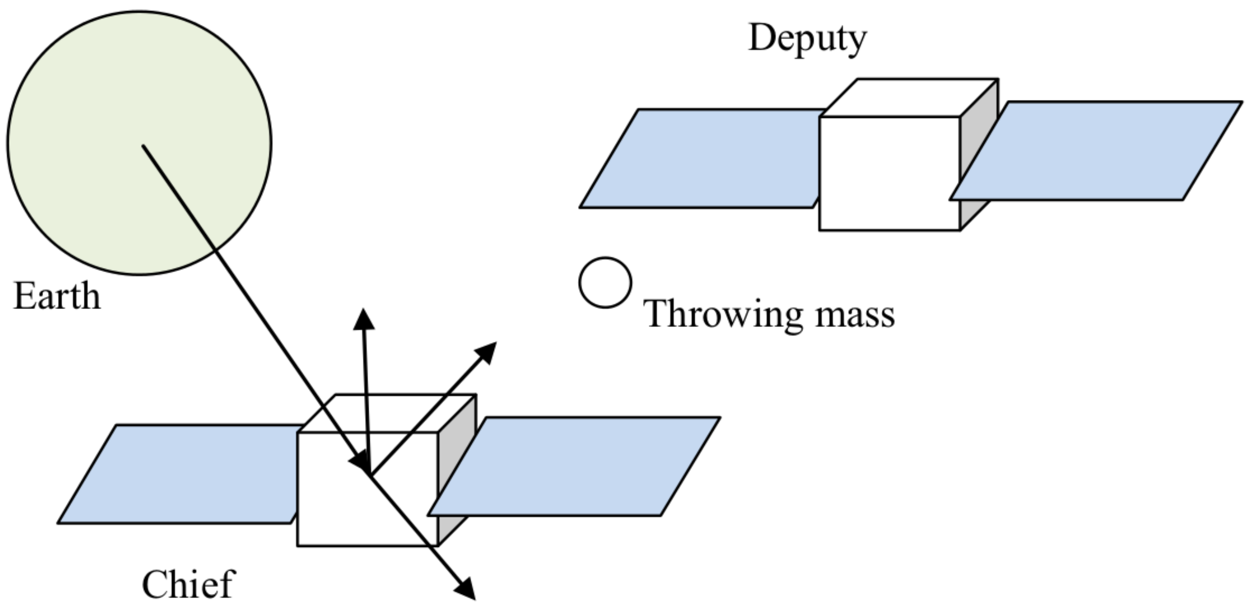
2006 – JC2Sat-FF project – two microsatellites, each of 18 kg, stacked in launch configuration.
The primary mission is to demonstrate FF technology using aerodynamic drag control by varying cross-sectional area of the spacecraft.
The advantage – absence of a propulsion system.



Methods of Control

Mass exchange control

- Mass as third body transfers momentum
- Solid body or drops or rays



Conclusion

- Models
  - Clohessy-Wiltshire
  - Eccentricity
  - Second-order nonlinearity
  - Gravitational perturbation from J2
- Control methods
  - Ion or colloid thrusters
  - Electrostatic/electromagnetic force
  - Aerodynamic drag
  - Mass exchange