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Relative motion control of two satellites by changing the reflective properties of the solar sail surface

Stepan Tkachev, Tatyana Petrova, Yaroslav Mashtakov

Keldysh Institute of Applied Mathematics

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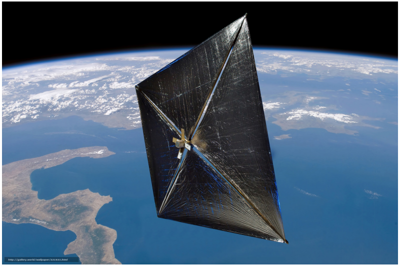
Problem statement

Problem: deployment and maintenance of the required relative orbit of two satellites.

Control source: the solar sails with the changeable reflective properties.



Assumptions: initial orbit of the leader satellite is circular. The follower satellite is moving along the orbit which is close to leader's one: $|\mathbf{r}_{\text{rel}}| \ll |\mathbf{r}_1|$. Disturbances include the solar radiation pressure and J_2 perturbation.

Motion equations (1)

Orbital motion

$$\ddot{\mathbf{r}} = -\mu_E \frac{\mathbf{r}}{r^3} + \mathbf{g},\ \mathbf{g} = \mathbf{f}_{J_2} + \mathbf{f}_s,\ \mathbf{f}_s = -\frac{\Phi_0 S}{c}(\mathbf{r}_s, \mathbf{n})((1-f)\mathbf{r}_s + 2f(\mathbf{r}_s, \mathbf{n})\mathbf{n})$$

Angular motion

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M}_{\text{control}} + \mathbf{M}_{\text{g}},\ \mathbf{M}_{\text{control}} = \int \mathbf{r} \times d\mathbf{F}_s,\ \mathbf{M}_{\text{g}} = 3\frac{\mu_E}{r^5}\mathbf{r} \times \mathbf{J}\mathbf{r}$$

the Hill-Clohessy-Wiltshire equations

$$\begin{cases} \ddot{x} + 2\omega\dot{z} = 0, \\ \ddot{y} + \omega^2 y = 0, \\ \ddot{z} - 2\omega\dot{z} - 3\omega^2 y = 0 \end{cases} \Rightarrow$$

perturbed equations

$$\begin{cases} \ddot{x} + 2\omega\dot{z} = u_x + g_x, \\ \ddot{y} + \omega^2 y = u_y + g_y, \\ \ddot{z} - 2\omega\dot{z} - 3\omega^2 y = u_z + g_z \end{cases}$$

Motion equations (2)

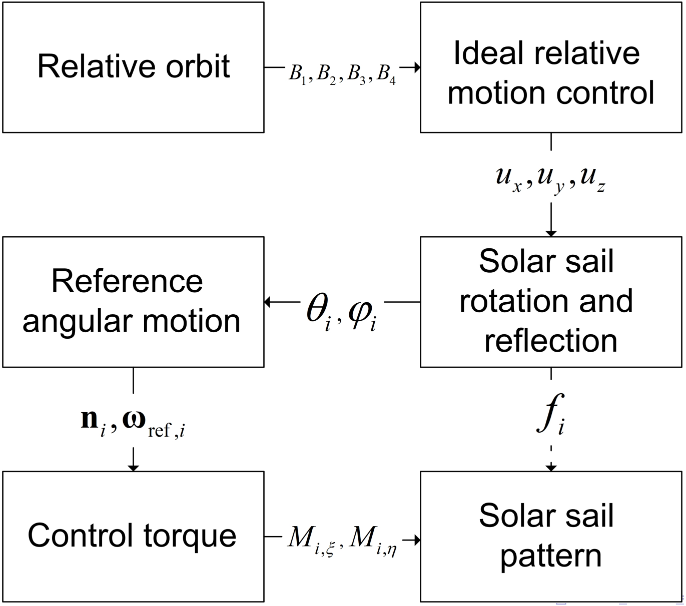
$$\begin{cases} x = -3C_1\omega t + 2C_2\cos\omega t - 2C_3\sin\omega t + C_4, \\ y = C_5\cos\omega t + C_6\sin\omega t, \\ z = 2C_1 + C_2\sin\omega t + C_3\cos\omega t \end{cases}$$

$$\Downarrow$$

$$\begin{aligned} x &= 2B_2\cos\psi_1 + B_3 & \dot{B}_1 &= \frac{1}{\omega}(u_x + g_x) \\ z &= B_2\sin\psi_1 + 2B_1 & \dot{B}_3 &= -3B_1\omega - \frac{2}{\omega}(u_z + g_z) \\ \dot{x} &= -2B_2\omega\sin\psi_1 - 3B_1\omega \quad \Leftrightarrow & \dot{B}_2 &= \frac{1}{\omega}((u_x + g_x)\cos\psi_1 - 2(u_x + g_x)\sin\psi_1) \\ \dot{z} &= B_2\omega\cos\psi_1 & \dot{\psi}_1 &= \omega - \frac{1}{B_2\omega}((u_z + g_z)\sin\psi_1 + 2(u_x + g_x)\cos\psi_1) \\ y &= B_4\sin\psi_2 & \dot{B}_4 &= \frac{1}{\omega}(u_y + g_y)\cos\psi_2 \\ \dot{y} &= B_4\omega\cos\psi_2 & \dot{\psi}_2 &= \omega - \frac{1}{\omega B_4}(u_y + g_y)\sin\psi_2 \end{aligned}$$

The aim: $B_1 = 0, B_3 = 0, B_2 = B_0, B_4 = 0$ — the relative motion is ellipse with the semi-axis equal to B_0 and $2B_0$.

Control synthesis



Relative motion control

1. Relative orbit stabilization
a. centre

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^2, \qquad \dot{V} = \frac{1}{\omega}B_1 u_x + B_3(-3B_1\omega - \frac{2}{\omega}u_z) \quad \Rightarrow \quad \begin{cases} u_x = -k_1 B_1, k_1 > 0, \\ u_z = \frac{1}{2}(-3B_1\omega^2 + k_2\omega B_2), k_2 > 0 \end{cases}$$

b. size

$$V = \frac{1}{2}B_1^2 + \frac{1}{2}B_3^2 + \frac{1}{2}(B_2 - B_0)^2 \quad \Rightarrow \quad \begin{cases} u_x = -k_3(B_1 - 2(B_2 - B_0)\sin\psi_1), k_3 > 0, \\ u_z = -k_4(-2B_3 + (B_2 - B_0)\cos\psi_1), k_4 > 0 \end{cases}$$

$$\dot{V} = \frac{1}{\omega}(B_1 - 2(B_2 - B_0)\sin\psi_1)u_x + \frac{1}{\omega}(-2B_3 + (B_2 - B_0)\cos\psi_1)u_z - 3B_1B_3\omega$$

The stability condition: $u_{\max} > |3B_1B_3\omega^2|$

2. The out-of-plane motion control

$$V = \frac{1}{2}B_4^2 \quad \Rightarrow \quad u_y = -k_y B_4 \cos\psi_2, k_y > 0$$

Relative motion control implementation

1. $$\begin{aligned} u_{x_s} &= 2Af_2\cos^2\theta_2\sin\theta_2\cos\varphi_2 - 2Af_1\cos^2\theta_1\sin\theta_1\cos\varphi_1,\\ u_{y_s} &= 2Af_2\cos^2\theta_2\sin\theta_2\sin\varphi_2 - 2Af_1\cos^2\theta_1\sin\theta_1\sin\varphi_1,\\ u_{z_s} &= A(1-f_2)\cos\theta_2 - A(1-f_1)\cos\theta_1 + 2Af_2\cos^3\theta_2 - 2Af_1\cos^3\theta_1 \end{aligned}$$

If θ_i is small, then

$$\begin{aligned} u_{x_s} &= 2Af_2\theta_2\cos\varphi_2 - 2Af_1\theta_1\cos\varphi_1,\\ u_{y_s} &= 2Af_2\theta_2\sin\varphi_2 - 2Af_1\theta_1\sin\varphi_1,\\ u_{z_s} &= Af_2 - Af_1, \end{aligned} \xrightarrow{(f_1-0.5)^2+(f_2-0.5)^2\to\min} \begin{aligned} f_1 &= 0.5 - \frac{u_{z_s}}{2A}\\ f_2 &= 0.5 + \frac{u_{z_s}}{2A}\\ 2f_{\min} - 1 &\le \frac{u_{z_s}}{A} \le 2f_{\max} - 1 \end{aligned}$$

Relative motion control implementation

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2. $L = (f_2\theta_2\cos\varphi_2 - f_1\theta_1\cos\varphi_1)^2 + (f_2\theta_2\sin\varphi_2 - f_1\theta_1\sin\varphi_1)^2 \to \max$, then $\left[\begin{array}{l}\varphi_1 = \varphi_2, \theta_1\theta_2 < 0,\\ \varphi_1 = \varphi_2 + \pi, \theta_1\theta_2 > 0\end{array}\right.$

Relative motion control implementation

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3. Let $\varphi_1 = \varphi_2$, then

$$\begin{cases} (f_2\theta_2 - f_1\theta_1)\cos\varphi = \dfrac{u_{x_s}}{2A};\\ (f_2\theta_2 - f_1\theta_1)\sin\varphi = \dfrac{u_{y_s}}{2A} \end{cases}$$

$\tan\varphi = \dfrac{u_{y_s}}{u_{x_s}}$. If $u_{x_s}\cos\varphi > 0$, then $f_2\theta_2 - f_1\theta_1 = \dfrac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}$.

Relative motion control implementation

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2. $L = (f_2\theta_2\cos\varphi_2 - f_1\theta_1\cos\varphi_1)^2 + (f_2\theta_2\sin\varphi_2 - f_1\theta_1\sin\varphi_1)^2 \to$ max, then $\left[\begin{array}{l}\varphi_1 = \varphi_2, \theta_1\theta_2 < 0,\\ \varphi_1 = \varphi_2 + \pi, \theta_1\theta_2 > 0\end{array}\right.$

3. Let $\varphi_1 = \varphi_2$, then

$$\begin{cases} (f_2\theta_2 - f_1\theta_1)\cos\varphi = \dfrac{u_{x_s}}{2A};\\ (f_2\theta_2 - f_1\theta_1)\sin\varphi = \dfrac{u_{y_s}}{2A} \end{cases}$$

$\tan\varphi = \dfrac{u_{y_s}}{u_{x_s}}$. If $u_{x_s}\cos\varphi > 0$, then $f_2\theta_2 - f_1\theta_1 = \dfrac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}$.

4. $L = \theta_1^2 + \theta_2^2 \to$ min, then $$\begin{cases} \theta_1 = -\dfrac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\dfrac{f_1}{f_1^2 + f_2^2},\\ \theta_2 = \dfrac{\sqrt{u_{x_s}^2 + u_{y_s}^2}}{2A}\dfrac{f_2}{f_1^2 + f_2^2} \end{cases}$$

Attitude control

Lyapunov control function

$$V = \frac{1}{2}(J_\xi \omega_{\text{rel},1}^2 + J_\eta \omega_{\text{rel},2}^2) + k_a \left(1 - \left(\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}, \mathbf{Bn}\right)\right)$$

$$\omega_{\text{rel}} = \omega - \omega_{\text{ref}},\ \boldsymbol{\omega}_{\text{ref}} = \mathbf{n} \times \dot{\mathbf{n}}$$

$$\dot{V} = \boldsymbol{\omega}_{\text{rel}}^{\mathrm{T}} \left(\mathbf{J}\dot{\boldsymbol{\omega}}_{\text{rel}} + k_a \mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}\right)$$

$$\mathbf{M}_{\text{control}} = \left(-k_\omega \omega_{\text{rel}} - \mathbf{M}_{\text{ext}} + \omega \times \mathbf{J}\omega - \mathbf{J}\omega \times \mathbf{B}\omega_{\text{ref}} + \mathbf{JB}\dot{\omega}_{\text{rel}} - k_a \mathbf{Bn} \times \begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}\right)_{\xi,\eta}$$

Solar sail pattern

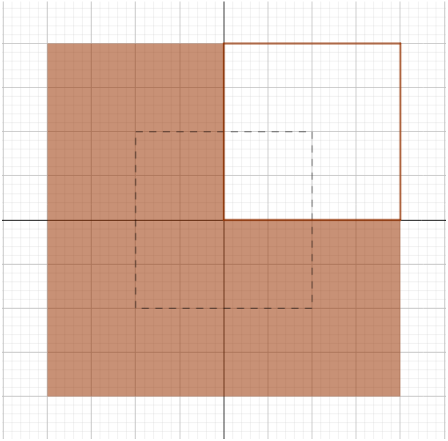
$$f = \frac{N}{n^2},\ \mathbf{M}_{\text{control}} = \int_S \mathbf{r} \times d\mathbf{F}_s$$

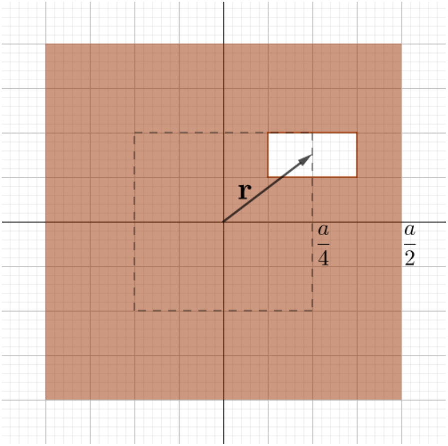
$$\mathbf{M}_{\text{control}} = \frac{1}{2}\left(\frac{a}{n}\right)^3 \cos^2\theta \begin{pmatrix} P \\ Q \\ -n\tan\theta\sin\alpha Q - \tan\theta\cos\alpha P \end{pmatrix}$$

$$P = 2I - (n+1)N,\ I = \sum_{(i,j):\alpha_{i,j}=1} i$$

$$Q = 2J - (n+1)N,\ J = \sum_{(i,j):\alpha_{i,j}=1} j$$

Solar sail cell pattern





Numerical examples

Orbit radius: $R_{\text{orb}} = 9000$ km

Initial relative orbit: $\mathbf{r}_{\text{rel}} = \begin{pmatrix}10 & 10 & 5\end{pmatrix}$ m, $\mathbf{v}_{\text{rel}} = \begin{pmatrix}0.05 & 0.1 & 0.1\end{pmatrix}$ m/s

Satellite mass: $m = 10$ kg

Sail: square, 5 m

Inertia tensor: $\mathbf{J} = \text{diag}\begin{pmatrix}2.1 & 2.1 & 3.8\end{pmatrix}$ kg·m^2

Inertial angular velocity:

$\omega_1 = \begin{pmatrix}0.002 & 0.003 & 0.001\end{pmatrix}$ rad/s

$\omega_1 = \begin{pmatrix}0.001 & 0.003 & 0.002\end{pmatrix}$ rad/s

Control parameters:

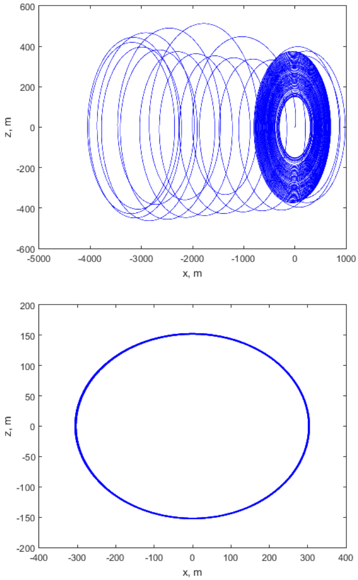
$k_1 = k_3 = k_4 = 20, k_2 = 10^{-6}$ s^{-1}

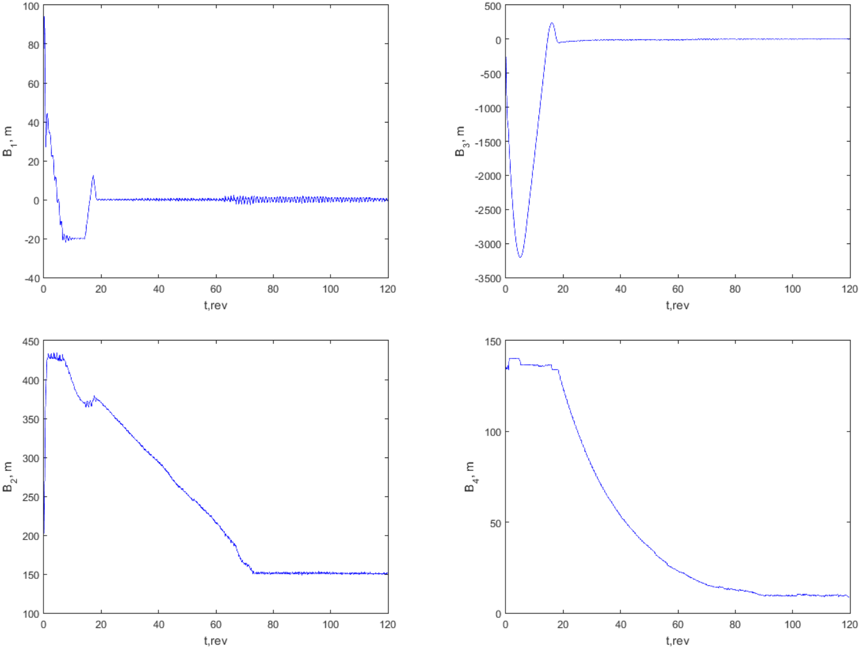
$k_\omega = 0.02$ N · m · s, $k_a = 10^{-4}$ N · m

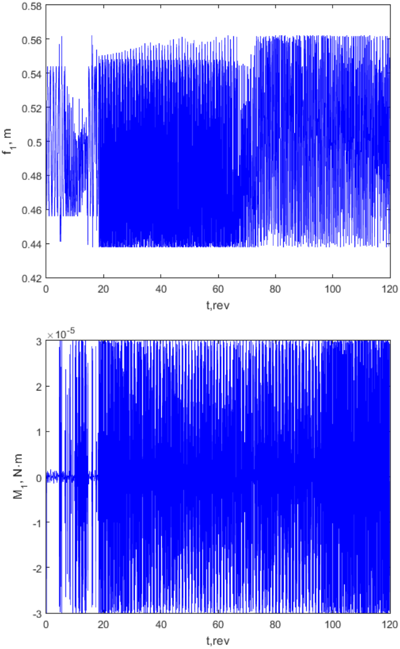
Maximum control force: $u_{\max} = 10^{-6}$ N

Maximum control torque: $M = 3 \cdot 10^{-5}$ N·m

Switch condition: $B_1 B_3 < 1$ m^2




100
80
60
40
20
0
-20
-40
B_1, m
0 20 40 60 80 100 120
t,rev
500
0
-500
-1000
-1500
-2000
-2500
-3000
-3500
B_3, m
0 20 40 60 80 100 120
t,rev
450
400
350
300
250
200
150
100
B_2, m
0 20 40 60 80 100 120
t,rev
150
100
50
0
B_4, m
0 20 40 60 80 100 120
t,rev


f_1, m
t,rev
θ_1, deg
t,rev
×10^-5
M_1, N·m
t,rev
×10^-5
M_2, N·m
t,rev

Conclusion

The scheme of two satellites formation flying control using the solar sail is proposed. It was shown that it is possible to control relative motion and corresponding attitude control using solar sail only.

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