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# MAGNETORQUERS ATTITUDE CONTROL FOR DIFFERENTIAL AERODYNAMIC FORCE APPLICATION TO NANOSATELLITE FORMATION FLYING CONSTRUCTION AND MAINTENANCE

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The paper considers a problem of satellites formation flying construction immediately after their launch. During the separation from the launcher some error in the ejection velocity is inevitable. It results in a slightly different orbital period of the satellites, so they will gradually move apart along the orbit and the relative trajectories become unbounded. The differential drag-based control is considered. The attitude control of the satellite is implemented by magnetorquers.

## INTRODUCTION

Nanosatellite formation flying is of a special interest for the scientific and commercial missions due to accesible possibility of the low-cost cluster launch as a piggy-back payload or using the launchers on the ISS. The multi satellite system is able to measure the spatial distribution of the near-Earth space parameters or to construct the robust remote sensing system. For the satellite formation flying on the Low-Earth-Orbit the most perspective control approach is to use the differential aerodynamic forces. It does not require propellant, the aerodynamic force acting on the satellite depends on its attitude, so to achieve the desirable relative motion the attitude control system (ACS) is required onboard the satellite. The most precise ACS is based on the reaction wheels, however they are affected to the saturation problem and require sufficient power supply that could be a problem for the nano and femto-satellites. In that paper the application of the ACS with magnetorquers is considered to obtain attitude for the implementation of required aerodynamic force. That type of the ACS is easier to use onboard of the nanosatellites, for attitude determination the magnetometer measurements only is processed.

The control approach based on the differential drag force was firstly proposed in 1980s by Leonard [1] under the assumption of a discrete change in the effective cross section of satellites flying in the group. He developed a control algorithm based on the proportional differential controller. A large number of papers applied a big variety of the different control algorithms using differential drag: PID regulator [2], linear-quadratic regulator [3], Lyapunov-based control [4,5], sliding mode control [6], optimal control [7] etc. However, all the mentioned above papers does not address the problem of the achieving the required attitude relative to the incoming airflow, though it is very crucial aspect especially if the ACS is magnetic.

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Magnetic control systems are widely used for satellite angular velocity damping and attitude stabilization. They are by far the cheapest and are among the most reliable, small and lightweight. The drawbacks are the worst accuracy and even underactuation. It is even possible to achieve three axis stabilization using the magnetorquers [8,9] [10]. Proper stabilization requires the real-time determination of the attitude motion. It is obtained by processing the attitude sensors measurements. For example, the three axis attitude control is available with the sole magnetometer and three magnetorquers for a CubeSat [10,11].

Due to the launch velocity errors the relative drifts is inevitable between the satellites, so the aerodynamic control is required to achieve bounded relative trajectories. The problem of the construction of the formation flying using magnetorquers is studied in the paper.

## PROBLEM STATEMENT

The problem of the satellite formation flying construction after their separation from the launcher is considered, i.e. the achievement of closed relative trajectories is required. Consider two nanosatellites launched in the low near-circular orbit. Each satellite is equipped with three orthogonal magnetorquers for attitude control and magnetometer for attitude determination. The relative motion is assumed to be known for both the satellite. This information can be obtained either via an inter-satellite link or using autonomous relative motion determination system (range finders, optical sensors, etc).

At the initial time the satellites move in accordance with the specified initial conditions after deploying from the launcher. The satellites deployment is carried out using a certain launch system (usually by special springs) with a certain execution error. In the absence of control it leads to a gradual increasing distances between the satellites. Due to the onboard magnetorquers the satellites relative translational motion can be controlled by the aerodynamic drag force which depends on the attitude of satellites relative to the incoming airflow. In the paper the 3U CubeSats are considered. They are the most popular nanosatellites nowadays and they have a form-factor quite proper for aerodynamic control because the ratio of the maximum to the minimum cross-sectional area is 3.

The main goal of the study is to investigate the possibility of the application of the three-axis attitude stabilization using magnetorquers to produce such a differential drag which leads the relative drift to zero. The possibility of formation flying construction with piecewise constant control depending on initial conditions is investigated. The effect of these parameters on the convergence rate is considered.

## MOTION EQUATIONS

### Free Motion Equations

Consider two satellites moving in close circular orbits. To describe the trajectories of satellites it is convenient to use the motion equations written in the relative reference frame. The general form of the equations of relative motion of any two satellites is rather complicated for analytical consideration, so at the initial stage of the study a simple motion model described by the Hill-Klochessi-Wiltshire equations is considered [1, 2]. The model describes the relative motion of two arbitrarily chosen satellites that fly in the central gravitational field of the Earth. In this model, an orbital reference frame is used, its origin (reference point) moves along a circular orbit of radius $r_0$ with an orbital angular velocity $\omega = \sqrt{\mu/r_0^3}$, where $\mu$ is the gravitational parameter of the Earth. The axis $Oz$ is directed from the center of the Earth, the axis $Oy$ is directed along the normal to the orbit plane, the axis $Ox$ complements the triplet (Fig. 1).

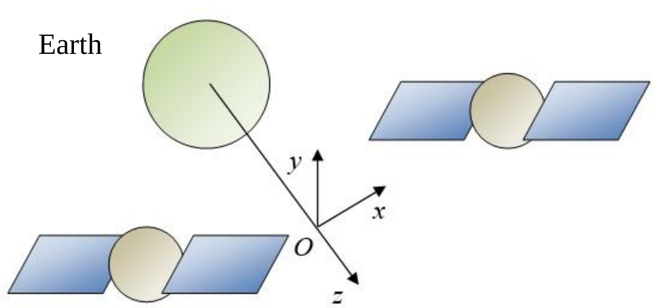


**Figure.1. The reference frame associated with the $O$ reference point, moving along a circular orbit**

Let $\mathbf{r}_1 = (x_1, y_1, z_1)$ and $\mathbf{r}_2 = (x_2, y_2, z_2)$ be the coordinates of the conditional first and second satellites in the reference frame. Then for the coordinates $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = (x, y, z)$ of the relative position vector of the satellites, one can write the following motion equations:

$$\begin{cases} \ddot{x} + 2\omega\dot{z} = 0, \\ \ddot{y} + \omega^2 y = 0, \\ \ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0, \end{cases}$$

The solution of the system is:

$$\begin{cases} x(t) = -3C_1\omega t + 2C_2\cos(\omega t) - 2C_3\sin(\omega t) + C_4, \\ y(t) = C_5\sin(\omega t) + C_6\cos(\omega t), \\ z(t) = 2C_1 + C_2\sin(\omega t) + C_3\cos(\omega t), \end{cases}$$

where $C_i$ are constants that depend on the initial conditions $x_0 = x(0),\ y_0 = y(0),\ z_0 = z(0),\ \dot{x}_0 = \dot{x}(0),\ \dot{y}_0 = \dot{y}(0),\ \dot{z}_0 = \dot{z}(0)$, as follows:

$$C_1 = \frac{\dot{x}_0}{\omega} + 2z_0, C_2 = \frac{\dot{z}_0}{\omega}, C_3 = -3z_0 - \frac{2\dot{x}_0}{\omega},$$
$$C_4 = x_0 - \frac{2\dot{z}_0}{\omega}, C_5 = \frac{\dot{y}_0}{\omega}, C_6 = y_0.$$

The term responsible for the relative drift: $-3C_1\omega t$. Thus, the relative trajectory of two satellites is closed if and only if $C_1 = 0$. However, in practice such ideal initial conditions can not be specified, and in the case of perturbations and nonlinear effects, there is always a relative drift between the satellites. Therefore, to achieve the bounded relative trajectory the satellites must be controlled for the drift elimination.

**Controlled Motion Equations**

Consider the application of the aerodynamic drag force for the group control. Since the drag force is directed against the satellite velocity vector and the group moves along near circular orbit it is assumed that the drag force is aligned with axis . The model of aerodynamic drag force $f$ acting on the satellite can be represented in the following form:

$$f = -\frac{1}{2}C_a \rho V^2 \Delta S \sin\alpha - \frac{1}{2}C_a \rho V^2 S_0,$$

where $C_a$ is the aerodynamic drag coefficient, $\rho$ is the density of the atmosphere, $V$ is the velocity of the incoming flow, $\Delta S$ is the difference between maximum and minimum value of the cross-sectional area of satellite, $S_0$ is the minimum value of the cross-sectional area of satellite, $\alpha \in [0; \pi/2]$ is the angle between the direction of the incoming airflow and longitudinal axis of satellites that assumed to be axisymmetric. The satellites in the group are supposed to be identical, so the values of the $\Delta S$, $S_0$, $C_a$ are the same for all the satellites. The velocity of the atmosphere due to Earth rotation is neglected and it is assumed that the velocity of the incoming flow for all the satellites is equal to the orbital velocity $V = \sqrt{\mu / r_0}$ . It is assumed that satellites are equipped with attitude control system, it allows them to change the angle $\alpha$ and thereby to control the value of the aerodynamic drag force. *Ox*

The difference between aerodynamic drag forces acting on the *1*-st and *2*-nd taking into account that the second term is equal for all of the satellites is as follows:

$$f = f_2 - f_1 = -\frac{1}{2}C_a \rho V^2 \Delta S \left(\sin\alpha_2 - \sin\alpha_1\right). \tag{1}$$

According to the differential drag model the force value is limited and the maximum value is as follows:

$$\max|f| = \frac{1}{2}C_a \rho V^2 \Delta S \ .$$

Consider a controlled motion equations of a swarm of satellites. Since the control is implemented using differential drag force the acceleration vector $\mathbf{u} = \mathbf{u}_2 - \mathbf{u}_1 = (u_x, u_y, u_z)$ have a non-zero component only along the axis $Ox$, i.e. $u_y = u_z = 0$. Let $u = u_x = f/m$, where $m$ is the mass of the satellite. Then the relative motion equations for satellites are as follows:

$$\begin{cases} \ddot{x} + 2\omega\dot{z} = u, \\ \ddot{y} + \omega^2 y = 0, \\ \ddot{z} - 2\omega\dot{x} - 3\omega^2 z = 0. \end{cases}$$

Assume that for the defined time interval $\Delta T$ the value of the control $u$ is constant. It means that during $\Delta T$ the attitude of the satellites do not change. Then the solution is:

$$\begin{cases} x(t) = -3C_1\omega t + 2C_2\cos(\omega t) - 2C_3\sin(\omega t) + C_4 + \dfrac{4u}{\omega^2} - \dfrac{3t^2 u}{2}, \\ y(t) = C_5\sin(\omega t) + C_6\cos(\omega t), \\ z(t) = 2C_1 + C_2\sin(\omega t) + C_3\cos(\omega t) + \dfrac{2tu}{\omega}. \end{cases} \tag{2}$$

The differential aerodynamic drag force has no effect on the motion along $Oy$ axis, it is defined only by the initial conditions after the launch. That is why planar motion of the satellites in $Oxz$ plane will be considered.

To eliminate the relative drift proportional to the constant $C_1$, it is necessary to achieve such an initial conditions that

$$\frac{\dot{x}(0)}{\omega} = -2z(0). \tag{3}$$

Assume that initially the equality is not satisfied. Let us find a constant control $u$ such that, at the time $\Delta T$ after the beginning of the controlled motion, the following desired equality for $x(\Delta T)$ and $z(\Delta T)$ will be satisfied:

$$\frac{\dot{x}(\Delta T)}{\omega} = -2z(\Delta T). \tag{4}$$

Substitute (2) into (4) we derive the following constant control value

$$u = \frac{-\omega C_1}{\Delta T} = \frac{-\omega}{\Delta T}\left(\frac{\dot{x}(0)}{\omega} + 2z(0)\right). \tag{5}$$

So, in the case of two satellites the constant control (5) leads to a closed relative trajectory. However, in the case if calculated control $u$ is more than maximum value of the differential drag acceleration $u_{\max}$ then one need to increase the time interval $\Delta T$ or to apply $u_{\max}$ several time intervals $\Delta T$ until the $C_1 = 0$ after the control application.

**Angular Motion Equations**

Rigid spacecraft angular motion is considered. The satellite is equipped with three mutually orthogonal magnetorquers and three axis magnetometer. Magnetorquers can produce any restricted dipole moment. Disturbing torques include gravitational and unknown ones. The latter are represented by constant and/or arbitrary Gaussian values. Inertia tensor knowledge is also erroneous.

Satellite attitude is represented using Euler angles $\alpha, \beta, \gamma$ (rotation sequence 2–3-1), direction cosines matrix $\mathbf{A}$ and its elements $a_{ij}$ (used for analytical study) and quaternion $\Lambda = (\mathbf{q}, q_0)$ (used for numerical simulation). Angular velocity may represent either absolute motion ($\boldsymbol{\omega}$ and its components $\omega_i$) or relative motion with respect to orbital reference frame ($\boldsymbol{\Omega}$ and $\Omega_i$). Absolute and relative velocities are related by

$$\boldsymbol{\omega} = \boldsymbol{\Omega} + \mathbf{A}\boldsymbol{\omega}_{orb} \tag{6}$$

where $\boldsymbol{\omega}_{orb} = (0, \quad \omega_0, \quad 0)$ is the orbital reference frame angular velocity.

Euler equations for the satellite with arbitrary inertia tensor $\mathbf{J} = diag(A, B, C)$ are

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M} \tag{7}$$

for absolute angular velocity and

$$\mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega} = \mathbf{M} + \mathbf{M}_{rel} \tag{8}$$

where $\mathbf{M}_{rel} = -\mathbf{J}\mathbf{W}_{\boldsymbol{\omega}}\mathbf{A}\boldsymbol{\omega}_{orb} - \boldsymbol{\Omega}\times\mathbf{J}\mathbf{A}\boldsymbol{\omega}_{orb} - \mathbf{A}\boldsymbol{\omega}_{orb}\times\mathbf{J}(\boldsymbol{\Omega}+\mathbf{A}\boldsymbol{\omega}_{orb})$ for relative angular velocity. $\mathbf{W}_{\mathbf{y}}$ is a skew-symmetric matrix for any $\mathbf{y}$,

$$\mathbf{W}_{\mathbf{y}} = \begin{pmatrix} 0 & y_3 & -y_2 \\ -y_3 & 0 & y_1 \\ y_2 & -y_1 & 0 \end{pmatrix}. \tag{9}$$

The torque may contain control part $\mathbf{M}_{ctrl}$ and disturbing part. The latter is divided into gravitational and unknown one, $\mathbf{M} = \mathbf{M}_{ctrl} + \mathbf{M}_{gr} + \mathbf{M}_{dist}$.

Dynamical equations are supplemented with kinematic relations. Quaternion kinematics is

$$\begin{aligned} \dot{\mathbf{q}} &= \frac{1}{2}(q_0\boldsymbol{\Omega} + \mathbf{W}_{\boldsymbol{\Omega}}\mathbf{q}), \\ \dot{q}_0 &= -\frac{1}{2}\mathbf{q}^T\boldsymbol{\Omega}. \end{aligned} \tag{10}$$

Euler angles are used for analytical analysis, in this case

$$\begin{aligned} \frac{d\alpha}{dt} &= \frac{1}{\cos\beta}(\Omega_2\cos\gamma - \Omega_3\sin\gamma), \\ \frac{d\beta}{dt} &= \Omega_2\sin\gamma + \Omega_3\cos\gamma, \\ \frac{d\gamma}{dt} &= \Omega_1 - tg\beta(\Omega_2\cos\gamma - \Omega_3\sin\gamma). \end{aligned} \tag{11}$$

Control torque is $\mathbf{M}_{ctrl} = \mathbf{m}\times\mathbf{B}$ where $\mathbf{m}$ is the dipole control moment of the satellite, $\mathbf{B}$ is the geomagnetic induction vector in bound reference frame. Consider control torque based on the PD-controller

$$\mathbf{m} = -k_{\omega}\mathbf{B}\times\boldsymbol{\Omega} - k_a\mathbf{B}\times\mathbf{S} \tag{12}$$

where $\mathbf{S} = \left(a_{23}-a_{32},\ a_{31}-a_{13},\ a_{12}-a_{21}\right)^T$. It provides necessary attitude [8,9,14]. Control parameters have decisive influence on the algorithm performance. They are adjusted manually in the vicinity of optimal ones obtained using Floquet theory [15].

Gravitational torque is

$$\mathbf{M}_{gr} = 3\omega_0^2(\mathbf{A}\mathbf{e}_3)\times\mathbf{J}(\mathbf{A}\mathbf{e}_3) \tag{13}$$

where $\mathbf{e}_3 = (0,\quad 0,\quad 1)$ is the satellite radius-vector in orbital frame.

Unknown disturbing torque is modelled using three different approaches. Gaussian distribution of the order of $5\cdot10^{-7}\,N\cdot m$ allows modelling arbitrary disturbances with small effect on satellite motion since control torque is few orders greater. Constant disturbance on the level of $10^{-7}\,N\cdot m$ augmented with Gaussian one represents more notable disturbance. Constant torque may arise due to aerodynamics or solar pressure acting on a satellite with vast solar panels. The worst case is constant torque of $5\cdot10^{-7}\,N\cdot m$ value.

Inclined dipole model is mainly used to represent geomagnetic field. It takes into account three first terms in a Gauss decomposition [16] and allows quite accurate field representation paired with simple computational procedures. Geomagnetic induction vector is

$$\mathbf{B} = \frac{\mu_e}{r^5}(\mathbf{k}r^2 - 3(\mathbf{kr})\mathbf{r})$$

where $\mathbf{k}$ is the Earth's dipole vector and $\mathbf{r}$ is the satellite radius-vector. Direct dipole model ($\mathbf{k}$ is antiparallel to Earth rotation axis) is used for analytical approaches, geomagnetic induction vector in orbital frame is

$$\mathbf{B}_{orb} = B_0 \begin{pmatrix} \cos u \sin i \\ \cos i \\ -2\sin u \sin i \end{pmatrix} \tag{14}$$

where $B_0 = \frac{\mu_e}{r^3}$, $\mu_e = 7.812 \cdot 10^6 km^3 \cdot kg \cdot s^{-2} \cdot A^{-1}$, $r$ is the satellite radius vector magnitude, $u$ is the argument of latitude, $i$ is the orbit inclination. Geomagnetic induction vector measurements are modelled as

$$\begin{aligned} \tilde{\mathbf{B}} &= \mathbf{A}\mathbf{B}_{orb} + \Delta\mathbf{B} + \boldsymbol{\eta}_{\mathbf{B}}, \\ \Delta\dot{\mathbf{B}} &= \boldsymbol{\eta}_{\Delta\mathbf{B}} \end{aligned} \tag{15}$$

where $\tilde{\mathbf{B}}$ are the magnetometer readings, $\mathbf{B}_{orb}$ is the modelled induction (inclined field is used in Kalman filter), $\Delta\mathbf{B}$ is magnetometer bias, $\boldsymbol{\eta}_{\mathbf{B}}$ and $\boldsymbol{\eta}_{\Delta\mathbf{B}}$ are Gaussian magnetometer error and bias rate of change, each with zero mean.

## NUMERICAL STUDY

Consider the application of the proposed control rules for the task of the nanosatellites formation flying construction after the launch. The scheme of the launch of the satellites is the same that used by PlanetLabs compan6 in 2017 for the launch of two 3U CubeSats from the launcher from ISS [17]. The photo of the launch is presented in Fig. 2. It is assumed that the satellites separate from the bus-launcher in the $Ox$ axis direction one after another with the time interval $\Delta t$ between the launches. The velocity of the ejection $V_e$ is the same for all the CubeSats, however due to launch system inaccuracy the ejection velocity $V_e$ is subjected to errors. So, the initial velocity vector $\mathbf{V}_0$ in orbital reference frame is modelled as follows:

$$\mathbf{V}_0 = \begin{bmatrix} V_e + \delta V \\ \delta V \\ \delta V \end{bmatrix}, \tag{16}$$

where $\delta V$ is ejection error, it is modelled as normally distributed random value with zero mean and covariance $\sigma_{\delta V}^2$.

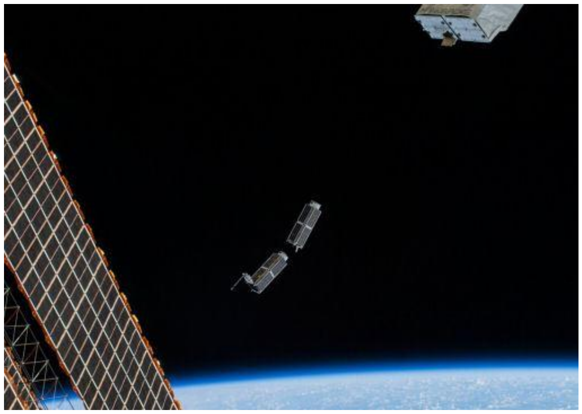

**Figure. 2. The photo of the launch of the two Doves PlanetLabs's 3U CubeSats from the ISS** [17]

All the parameters used for the simulation of the controlled motion of the swarm of the CubeSats are presented in Table 1. The constant atmosphere density is chosen in accordance to average atmosphere density along the orbit with 340 km altitude according to the Russian GOST model of upper atmosphere [18].

**Table 1. Parameters of simulation**

| | |
|---|---|
| Launch parameters | |
| Time interval between the launches, $\Delta t$ | 10 s |
| Ejection velocity, $V_e$ | 0.5 m/s |
| Ejection error deviation, $\sigma_{\delta V}$ | 0.015 m/s |
| CubeSats parameters | |
| Mass of satellite, $m$ | 3 kg |
| Difference between maximum and minimum value of the cross-sectional area, $\Delta S$ | 0.02 $\text{m}^2$ |
| Aerodynamic drag coefficient, $C_a$ | 2 |
| Inertia tensor, $J$ | $diag(1.2, 1.1, 0.5)\cdot 10^{-2}\,\text{kg}\cdot\text{m}^2$ |
| Magnetorquers maximum dipole moment, $\mathbf{m}_{\max}$ | 0.3 $\text{A}\cdot\text{m}^2$ |
| Magnetometer mean-square deviation, $\sigma_{\mathbf{B}}$ | 300 nT |
| Initial angular velocity, $\boldsymbol{\omega}_0$ | $[\,1; 1; 1\,]$ deg/s |
| Aerodynamic drag force parameters | |
| Constant atmosphere density, $\rho$ | $10^{-11}\,\text{kg/m}^3$ |
| Orbit altitude, $h$ | 340 km |
| Airflow velocity, $V=\sqrt{\mu/(R_E+h)}$ | 7.69 km/s |

The whole scheme of the control loop is presented in Figure 1. First the initial conditions are defined for the integration of the relative translational motion equations (2) and angular motion equations (7) for both the satellites. Then using the current state vector of relative translational motion the relative drift is calculated and using (5) the required control value is obtained. This value of the required differential drag force can be achieved if the required attitude of both the satellites is implemented. Using the drag model (1) the required angles relative to the incoming airflow is calculated. After that the magnetorquer control is aimed to stabilize according to the required attitude. During stabilization the actual attitude is used for the actual aerodynamic force calculation and for relative motion equations integration. The update of the required control for relative drift elimination is calculated with a certain time interval.

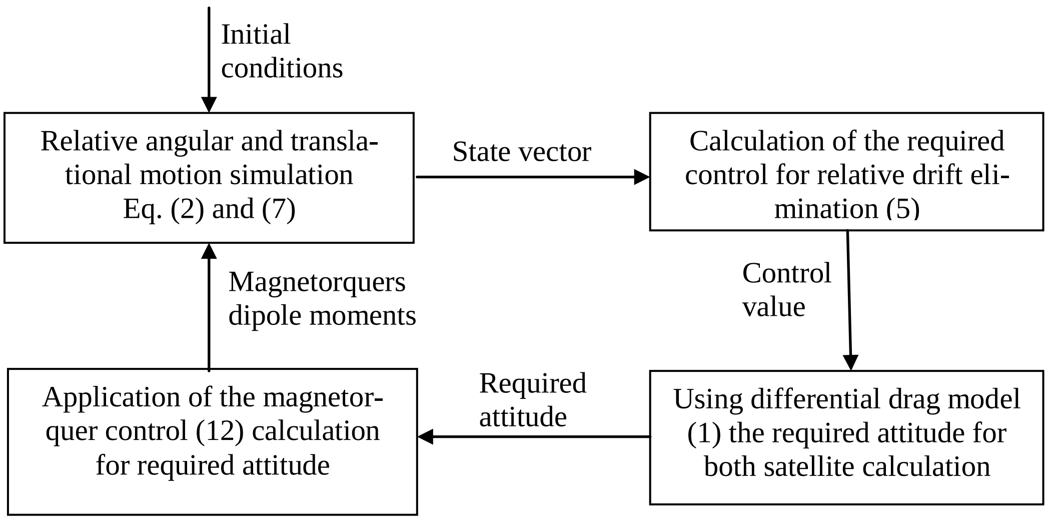


**Figure 3. Scheme of the control loop**

Consider the example of the relative motion simulation after the launch. The free uncomtrolled motion according to the initial conditions with ejection velocity errors is presented in Figure 4. Since the motion along the Oy axis is uncontrolled by the differential drag force, motion in the Oxz plane only is considered. As one can see in the Figure 4 the relative motion is unbounded, and the satellites are flying apart. Figure 5 demonstrates the controlled relative motion under the application of the proposed scheme. The trajectories become bounded in time and the relative drift converges to a zero as presented in Figure 6. Figure 7 shows the required angles relative to the incoming airflow and its actual values for two satellites. Initially the calculated control was more than maximum possible value that could be implemented by differential drag. That is why the required attitude so to align the longitudinal axis with velocity vector (i.e. $\alpha_1 = 0$) for the first satellite and to make that axis perpendicular to the incoming airflow for the second satellite ($\alpha_2 = 90^{\circ}$). It took several hours to stabilize along the required axes with accuracy about several degrees. Then the calculated control become less then the maximum value, so to implement it another attitude angle was required. The accuracy of the realization of required angles was about 20 degrees, but nonetheless the relative drift converges to a zero after 6 hours. Figure 8 demonstrates the values of the magnetic dipole moments of the magnetorquers during the considered example.

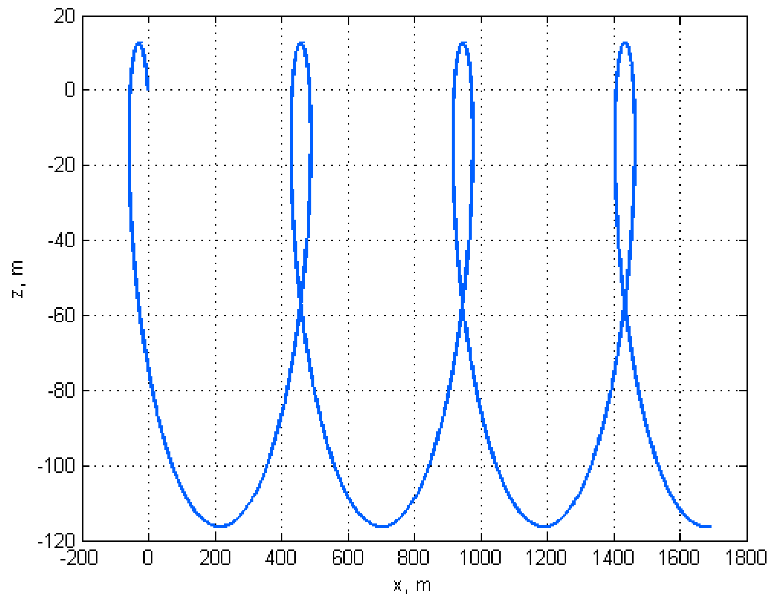


**Figure 4. Free relative motion after the launch**

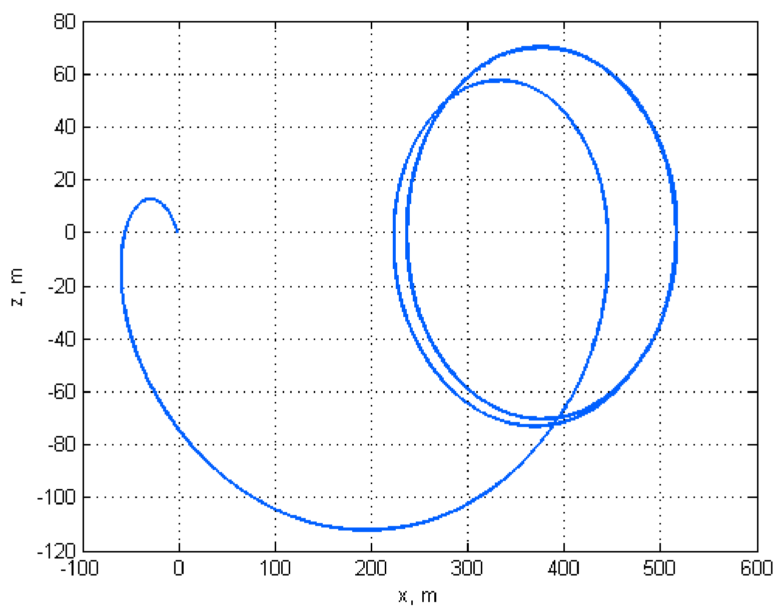


**Figure 5. Controlled relative motion after the launch**

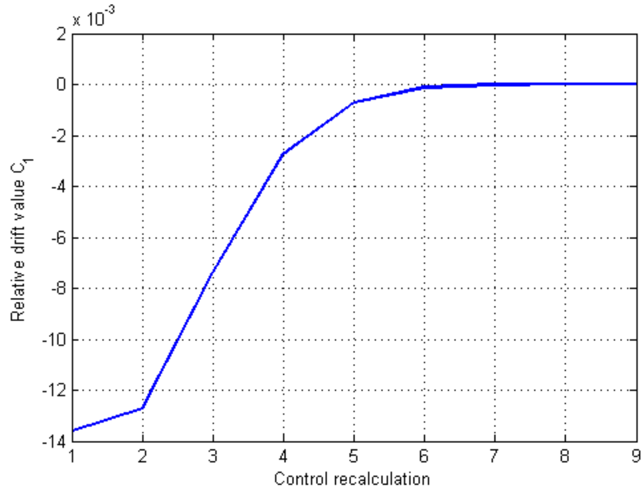


**Figure 6. The relative drift**

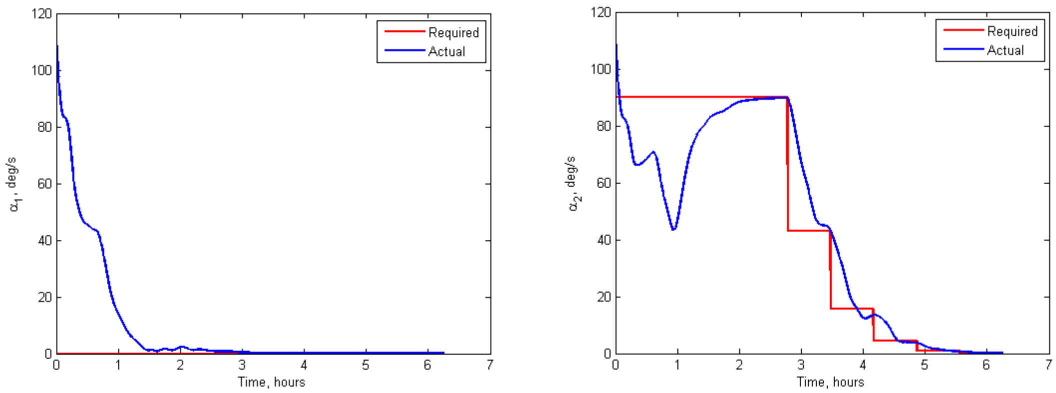


**Figure 7. The required and actual angles relative to the incoming airflow for two satellites**

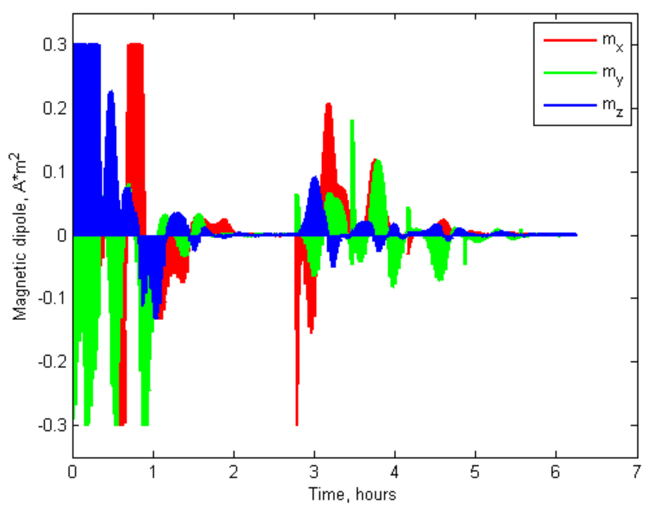


**Figure 8. The magnetorquers control values for the second satellite**

## CONCLUSION

The possibility of the magnetorquer attitude control application for formation flying contraction is demonstrated. Despite the low accuracy of the satellites stabilization relative to the incoming airflow the relative drift converges to a zero and the relative trajectories become bounded. In the future work the influence of the disturbances caused by harmonic $J_2$ and atmosphere density uncertainties will be investigated. Also the application of the magnetic control for the achievement of required relative trajectories are of special interest.


## ACKNOWLEDGMENTS

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