Algorithms of satellite formation flying control using aerodynamic drag

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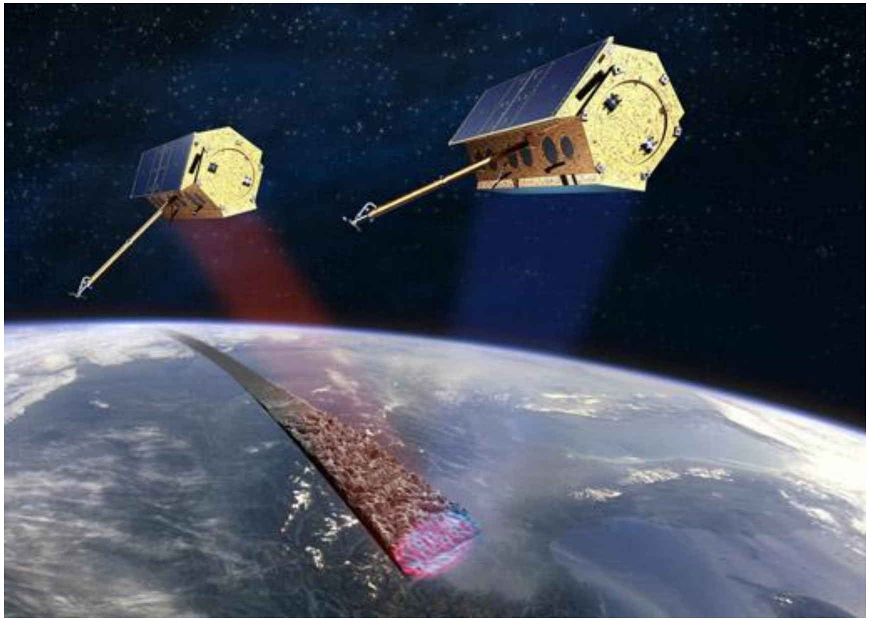
Satellite formation flying

A satellite formation consists of two of more satellites flying in a specified geometry to achieve a given mission

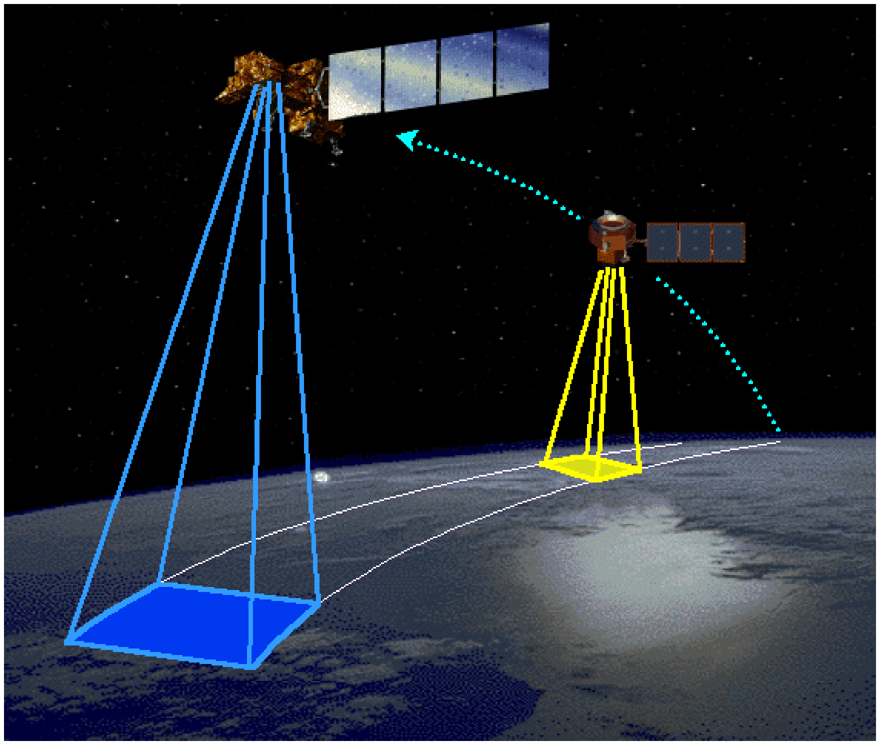
- more reliable
- cheaper
- more flexible



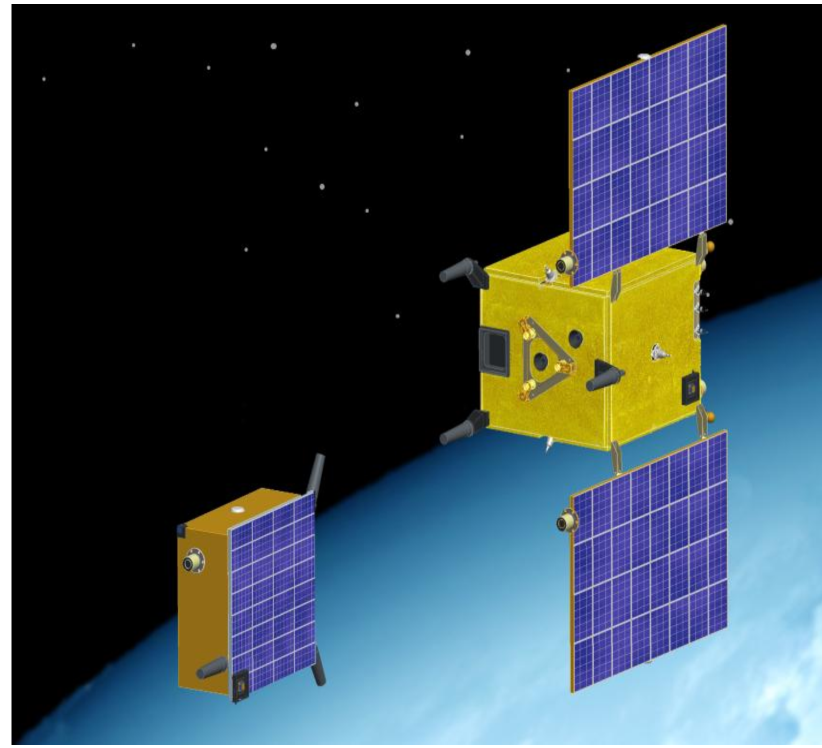
TanDEM-X
Earth Observation Satellites

«AeroCube» formation

Satellite formation flying



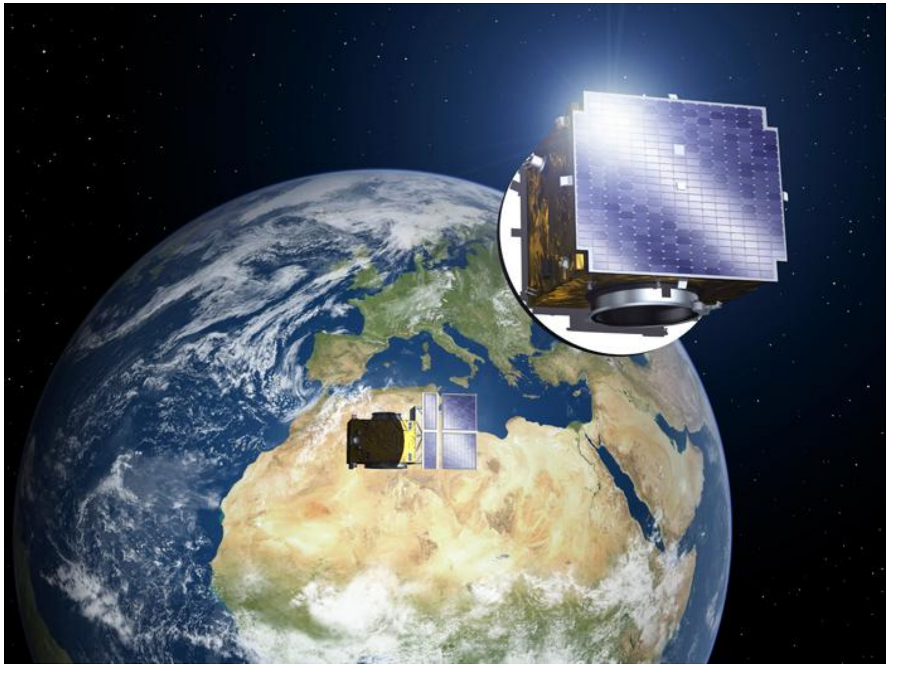
Landsat-7 being trailed by EO-1 covering the same area at different times



PRIZMA mission:
satellites Tango and Mango

Methods of satellite formation flying control

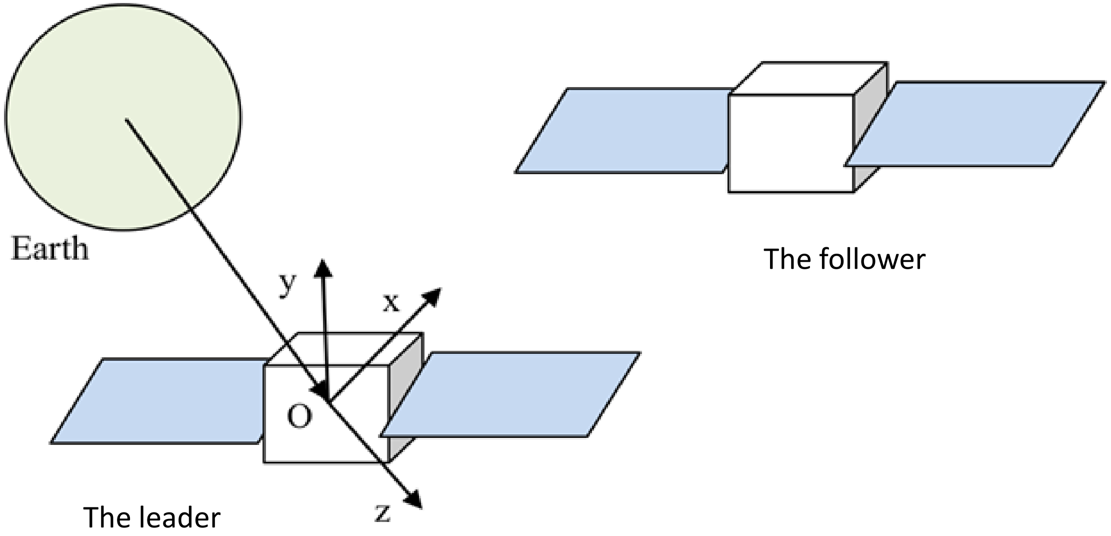
- Thrusters
- Electrostatic force
- Lorentz force
- Magnetorquers
- Aerodynamic drag
- Mass exchange



Proba-3's pair of satellites

Aerodynamic (AERO) drag control

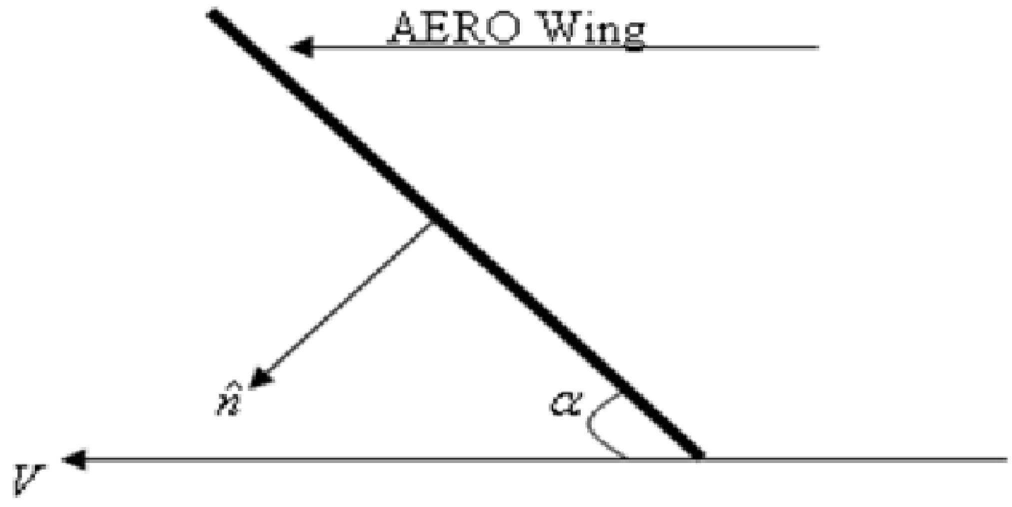
- There are two satellites: a leader and a follower
- Both satellites are equipped with AERO flaps
- The relative trajectory changes by rotation of AERO flaps



Relative Motion

HCW-equations

$$\begin{cases} \ddot{x} = -2\dot{z}\omega + f_x, \\ \ddot{y} = -y\omega^2, \\ \ddot{z} = 2\dot{x}\omega + 3z\omega^2, \end{cases}$$



AERO drag force

$$f_x = \frac{1}{2m}\rho C V^2 S \sin\Delta\alpha,$$

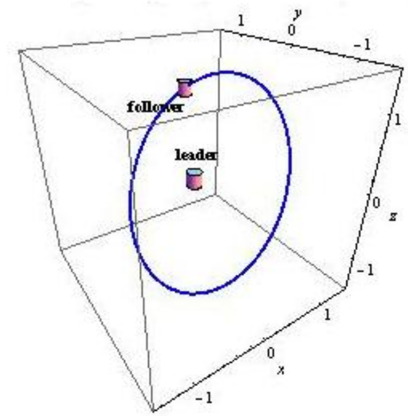
$$\Delta\alpha = \arcsin(\sin\alpha_1 - \sin\alpha_2),$$

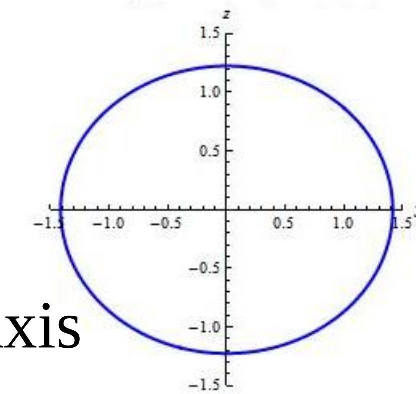
$$\alpha_1\alpha_2 = 0.$$

Relative Motion

The Solution of HCW-equations, if f_x=0

$$\begin{cases} x(t) = -3c_1\omega t + 2c_2\cos\omega t - 2c_3\sin\omega t + c_4, \\ y(t) = c_5\sin\omega t + c_6\cos\omega t, \\ z(t) = 2c_1 + c_2\sin\omega t + c_3\cos\omega t, \end{cases}$$





If $c_1 = 0$ the trajectory is closed

$\sqrt{c_2^2 + c_3^2}$ is proportional to the anplitude along x-axis and z-axis

$\sqrt{c_5^2 + c_6^2}$ is proportional to the anplitude along y-axis

Relative Motion

HCW-equations in state vector form for in-track and radial direction:

$$\dot{\mathbf{x}} = \mathrm{A}\mathbf{x} + \mathbf{b}u,$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ z \\ \dot{z} \end{bmatrix} \quad \mathrm{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2\omega \\ 0 & 0 & 0 & 1 \\ 0 & 2\omega & 3\omega^2 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ \mathrm{B}_x \\ 0 \\ 0 \end{bmatrix},$$

$$\mathrm{B}_x = -\frac{1}{2}\rho C V^2 S, \quad u = \sin\alpha_2 - \sin\alpha_1.$$

AERO drag force has an affect only on in-track and radial direction.

Optimal Control

The state equation:

$$\dot{\mathbf{x}} = \mathrm{A}\mathbf{x} + \mathbf{b}u$$

The boundary conditions:

$$\phi[\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f] = 0$$

The aim of control is to minimize the cost functional

$$J = \Phi[\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f] + \int_{t_0}^{t_f} L(\mathbf{x}(t), u(\mathrm{t}))\,\mathrm{dt}$$

+

- Robust
- Minimizing the energy lost
- Reaching the aim in finite time

-

- Difficultly implemented
- Discontinuous

LQR

Control force

$$\mathbf{u} = \mathrm{K}\mathbf{e},$$

where $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$

minimize the following functional

$$J = \int_0^\infty (\mathbf{e}^T \mathrm{Q}\mathbf{e} + \mathbf{u}^T \mathrm{R}\mathbf{u})\, dt,$$

where Q,R – positive-definite matrix.

Feedback control

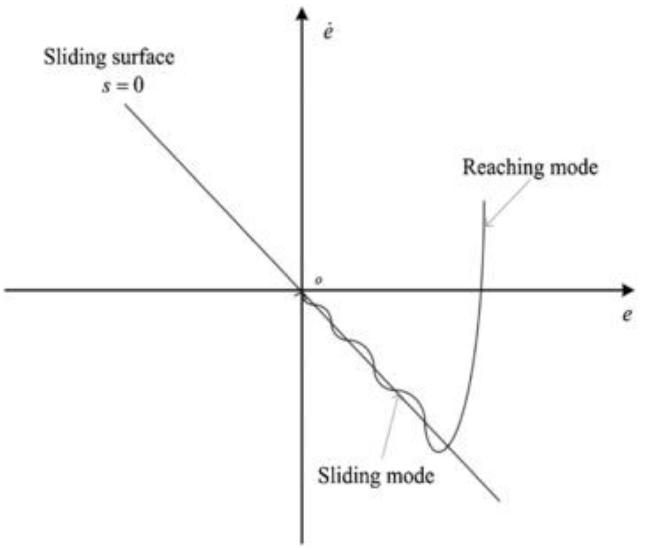
$$\mathbf{u} = -\mathrm{R}^{-1}\mathrm{B}^T \mathrm{P}\mathbf{e},$$

here matrix P is solution of the Riccati equation

$$\mathrm{Q} - \mathrm{PBR}^{-1}\mathrm{B}^T\mathrm{P} + \mathrm{PA} + \mathrm{A}^T\mathrm{P} = 0.$$

Sliding Mode Control (SMC)

- SMC is a nonlinear control method that alters the dynamic of a system.
- Control signal forces the system to "slide" along a cross-section of the sliding surface, until the system reaches the required surface.



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- Easily implemented
- Robust
- Reaching sliding mode in finite time

-

- Discontinuous
- Depending on sliding surface
- Not minimizing the energy lost

Sliding Mode Control

The Lyapunov fuction of the system is defined as:

$$V = \frac{1}{2}S^2, \text{ where } S = \dot{e}_x + K_1 e_z + K_2 \dot{e}_z + K_3 e_x.$$

If the system is stable

$$\dot{V} = S\dot{S} < 0, \text{ therefore lets assume } \dot{S} = -n\,\text{sign}(S), \text{ where n=const.}$$

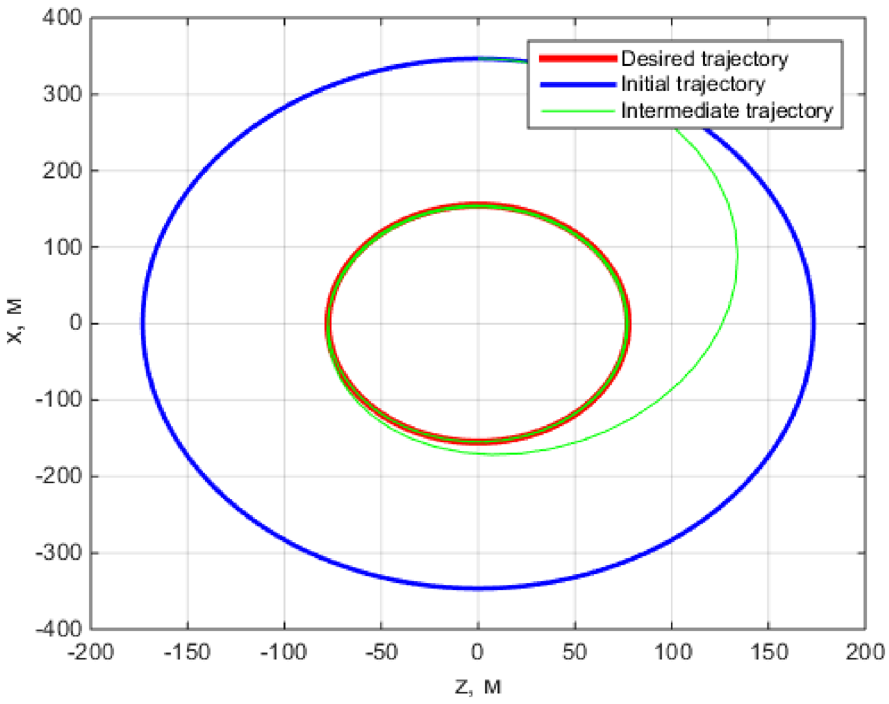
Taking into consideration

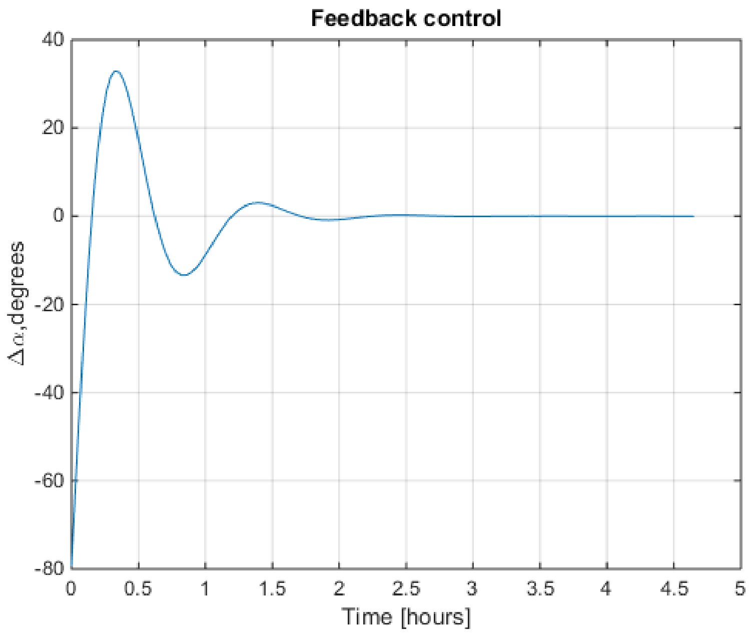
$$\ddot{e}_x = [\ddot{x} - \ddot{x}_d], \ddot{x} = -2\dot{z}\omega + u,$$

Feedback control is

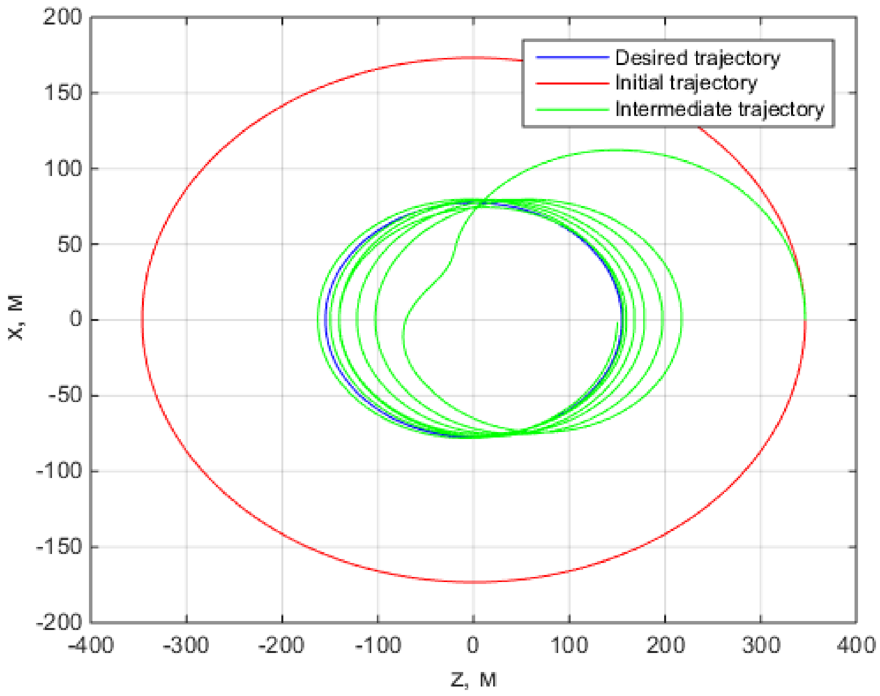
$$u = -K_1 \dot{e}_z - K_2 \ddot{e}_z - K_3 \dot{e}_x - nsign(S) + 2\dot{z}\omega + \ddot{x}_d.$$

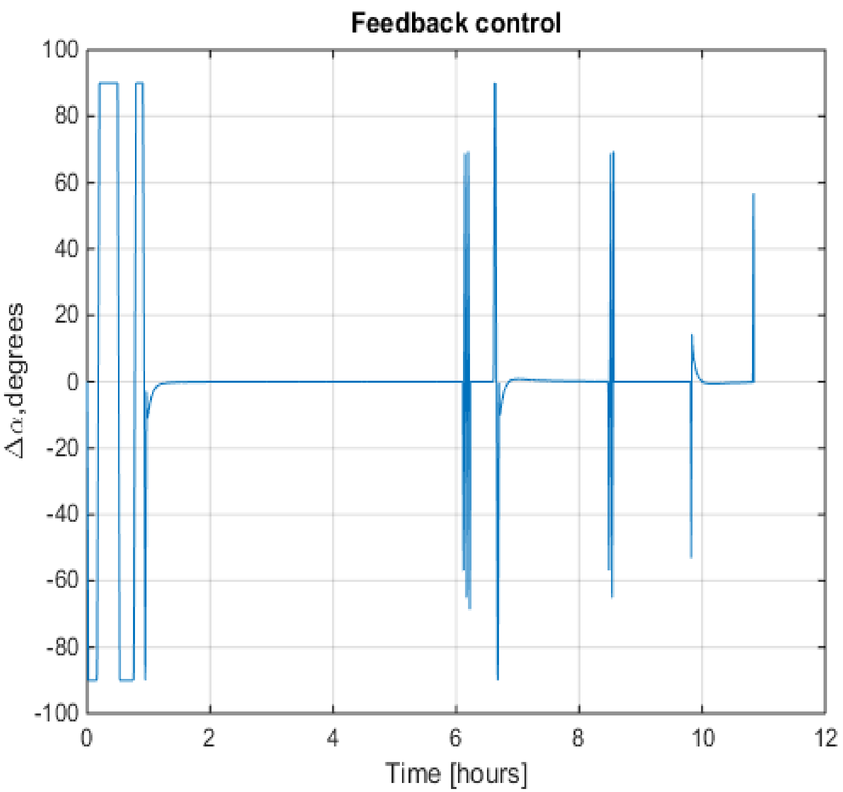
Numerical Simulation
LQR





Numerical Simulation
Sliding Mode Control





Collision avoidance

Collisions can arise from
- A foreign object
- Other satellites in formation

Consider a avoidance region, in which satellite can not enter

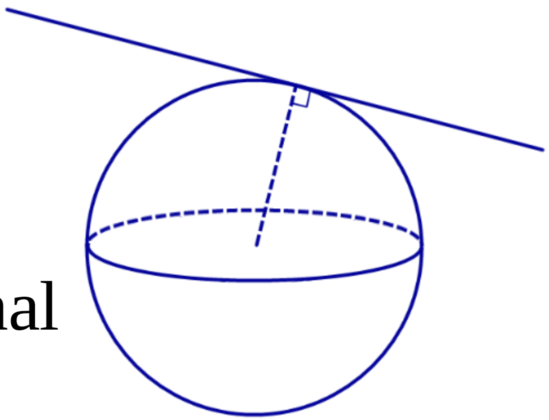
The problem is to find a appropriate trajectory, which be a tangent of the avoidance region, using AERO control

Iridium 33 and Kosmos-2251 collided in 2009

Optimal control

Assume that the a avoidance reqion is a schere (radius R)

$$\begin{cases} x(T)^2 + z(T)^2 - R^2 = 0, \\ x(T)\dot{x}(T) + z(T)\dot{z}(T) = 0, \end{cases}$$



The contol also minimizes the functional

$$J = \int_{t_0}^{T} (u^2(t))dt.$$

The Pontryagin Maximum Principle

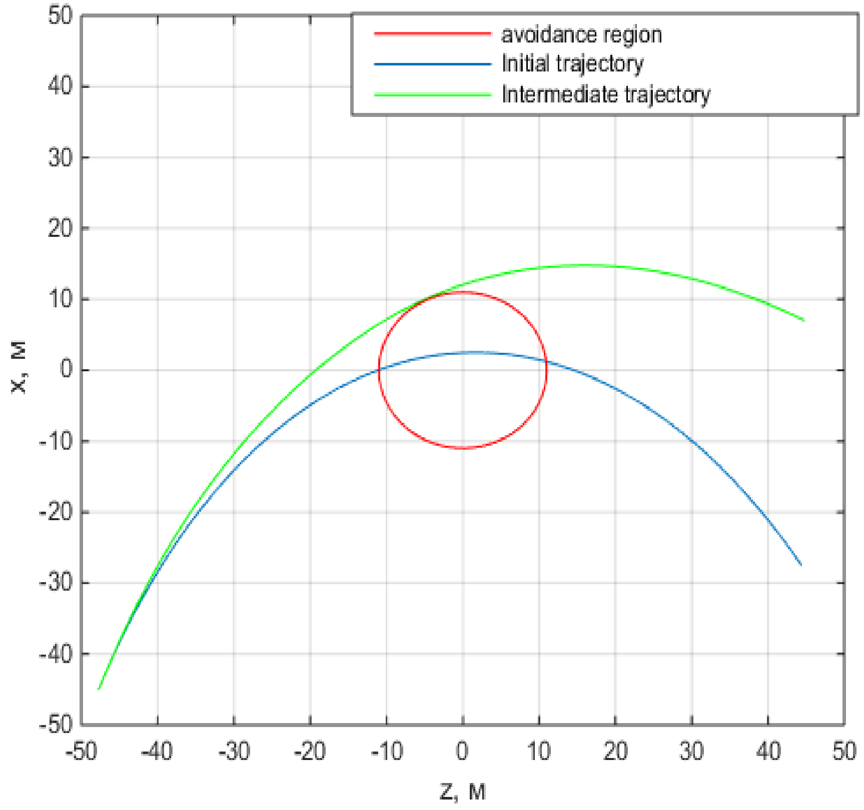
Hamiltonian of the system

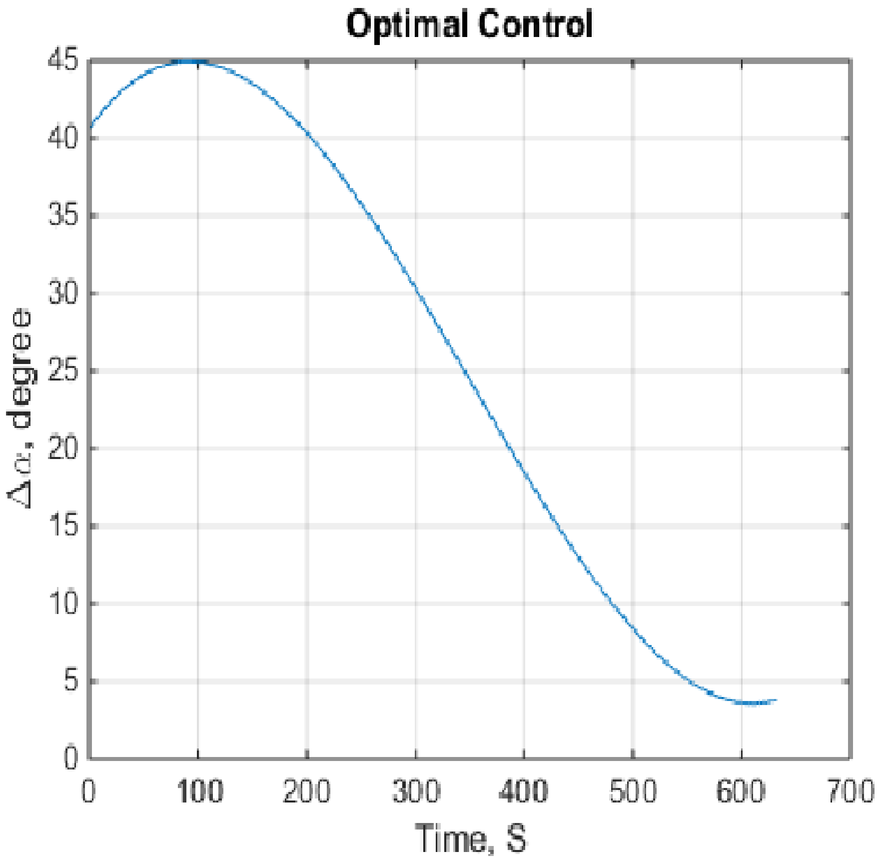
$$\tilde{H} = -\frac{1}{2}u^2 + \psi_1\dot{x} + \psi_2(u - 2\dot{z}\omega) + \psi_3\dot{z} + \psi_4(2\dot{x}\omega + 3z\omega^2)$$

Solving a system of differential equations, the control can be founded

$$\begin{cases} \dfrac{\partial \tilde{H}}{\partial u} = 0, \\ \dfrac{d\psi_j(t)}{dt} = \dfrac{\partial \tilde{H}}{\partial x_j}, \\ \dfrac{dx_j(t)}{dt} = -\dfrac{\partial \tilde{H}}{\partial \psi_j}, \\ \mathbf{x}(t_0) = \mathbf{x}_0, \\ \mathbf{x}(T) = \mathbf{x}_T, \\ \tilde{H}(T) = 0 \end{cases}$$

The Pontryagin Maximum Principle





Conclusion

Algorithms of satellite formation flying control using aerodynamic drag

- LQR
- Sliding mode control
- The Pontryagin Maximum Principle