2th IAA Conference on Dynamics and Control of Space Systems

Advanced Technique for Kalman Filter Adjustment and Its Implementation Onboard of "TabletSat" Microsatellite Series

Danil Ivanov, Michael Ovchinnikov
Keldysh Institute of Applied Mathematics of RAS
Nikita Ivlev, Stanislav Karpenko
“SputniX” Ltd.

Content

- **Introduction**
- **Kalman Filter Adjustment Technique**
- **TabletSat Details**
- **Examples of Application**
- **Conclusion**

Algorithms based on Kalman filter

The satellite motion model

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},t) + \mathbf{q}_k,$$

$$\mathrm{M}(\mathbf{q}_k) = 0,\ \mathrm{M}\left(\mathbf{q}_k\mathbf{q}_k^T\right) = Q_k,$$

The measurement model

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, t) + \mathbf{r}_k.$$

$$\mathrm{M}(\mathbf{r}_k) = 0,\ \mathrm{M}\left(\mathbf{r}_k\mathbf{r}_k^T\right) = R_k.$$

Linearization

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}(\mathbf{x},t)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^-}, \mathbf{H}_k = \left.\frac{\partial \mathbf{h}(\mathbf{x},t)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^-}$$

$$\mathbf{\Phi}_k = \mathbf{E} + \mathbf{F}_k\left(t_k - t_{k-1}\right).$$

Prediction stage

1. Project the state ahead

$$\hat{\mathbf{x}}_k^- = \hat{\mathbf{x}}_{k-1}^+ + \int_{t_{k-1}}^{t_k} \mathbf{f}(\mathbf{x},t)dt,$$

2. Project the error covariance ahead

$$\mathbf{P}_k^- = \mathbf{\Phi}_k \mathbf{P}_{k-1}^+ \mathbf{\Phi}_k^{\mathrm{T}} + \mathbf{Q}_k,$$

Correction stage

1. Compute the Kalman Gain

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^{\mathrm{T}} (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1},$$

2. Update the estimate via $\mathbf{z}_k$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_k^-, t)],$$

3. Update the error covariance

$$\mathbf{P}_k^+ = [\mathbf{E} - \mathbf{K}_k \mathbf{H}_k]\mathbf{P}_k^-.$$

Initial state $\mathbf{x}_0$,

Initial error covariance $\mathbf{P}_0$, $\mathbf{Q}$, $\mathbf{R}$

The problem of EKF adjustment

- The real dynamic and measurement models differ from the ones used in EKF.
- Then the dynamical model error matrix Q and measurement model error matrix R become the adjustment parameters.
- The problem is to choose Q and R to increase the EKF estimation accuracy.

Kalman Filter Adjustment Techniques

Investigation technique	Shortcomings
Monte-Karlo simulations (*Mortari D., Markley L.S., Steyn W.H., Matthews O., Maybeck P.*)	Time-consumable
Genetic algorithms (*Bar-Itzhack, Oshman Y.*, *Clements R.*)	Time-consumable, obtain local minimum only
Adaptive Filtering *(Bekir E., Ng A., Kumar K.)*	Require more computational power
Covariance matrix computation for stationary system (Parusnikov N., *Golovan A., Balacrishnan A.)*	Applicable only for stationary system

We propose the advanced analytical adjustment technique for Kalman Filter performance investigation

Accuracy investigation for quasi stationary motion

For stationary system:

$$\mathbf{\Phi}_k = \mathbf{\Phi} = const,\ \mathbf{H}_k = \mathbf{H} = const,\ \mathbf{Q}_k = \mathbf{Q} = const,\ \mathbf{R}_k = \mathbf{R} = const,$$

the asymptotic error matrix $\mathbf{P}_\infty$ after convergence is existing and can be obtained from equation

$$\mathbf{P}_\infty = \left\{ \mathbf{E} - \left(\mathbf{\Phi}\mathbf{P}_\infty\mathbf{\Phi}^T + \mathbf{Q} \right) \mathbf{H}^T \left[\mathbf{H} \left(\mathbf{\Phi}\mathbf{P}_\infty\mathbf{\Phi}^T + \mathbf{Q} \right) \mathbf{H}^T + \mathbf{R} \right]^{-1} \mathbf{H} \right\} \left(\mathbf{\Phi}\mathbf{P}_\infty\mathbf{\Phi}^T + \mathbf{Q} \right).$$

Assumptions:

- The motion is slow, matrices $\mathbf{\Phi}$, $\mathbf{H}$ are close to be constant during sample time.
- The EKF converge at the particular matrices $\mathbf{\Phi}$ and $\mathbf{H}$, and one can calculate $\mathbf{P}_\infty$.
- The diagonal elements of $\mathbf{P}_\infty$ is an accuracy estimation.

Investigation of Influence of Unaccounted in EKF Perturbations

	Kalman filter equations	"Truth model" equations
Dynamical model	$\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{q}_k,\ \mathrm{M}(\mathbf{q}_k \mathbf{q}_k^T) = \mathbf{Q}_k,$	$\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{y}_{I,k},$ $\mathbf{y}_{I,k+1} = \mathbf{\Gamma}_{I,k} \mathbf{y}_k + \mathbf{\theta}_{I,k},$
Measurement model	$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k,\ \mathrm{M}(\mathbf{r}_k \mathbf{r}_k^T) = \mathbf{R}_k$	$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{y}_{II,k},$ $\mathbf{y}_{II,k+1} = \mathbf{\Gamma}_{II,k} \mathbf{y}_{II,k} + \mathbf{\theta}_{II,k}$
Prediction stage	$\hat{\mathbf{x}}_{k+1}^- = \mathbf{\Phi}_k \hat{\mathbf{x}}_k^+$	$\hat{\mathbf{x}}_k^+ = (\mathbf{E} - \mathbf{K}_k \mathbf{H}_k)\hat{\mathbf{x}}_k^- - \mathbf{K}_k \mathbf{y}_{II,k},$ $\mathbf{y}_{I,k}^+ = \mathbf{y}_{I,k}^-,$ $\mathbf{y}_{II,k}^+ = \mathbf{y}_{II,k}^-$
Correction stage	$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-)$	$\mathbf{x}_{k+1}^- = \mathbf{\Phi}_k \mathbf{x}_k^+ + \mathbf{y}_{I,k}^+,$ $\mathbf{y}_{I,k+1}^+ = \mathbf{\Gamma}_{I,k} \mathbf{y}_{I,k}^- + \mathbf{\theta}_{I,k},$ $\mathbf{y}_{I,k+1}^- = \mathbf{\Gamma}_{II,k} \mathbf{y}_{II,k}^- + \mathbf{\theta}_{II,k}$

Calculation of the error matrix P for the extended state vector:

$$\mathbf{\xi} = (\mathbf{x}, \mathbf{y}_I, \mathbf{y}_{II})^T,$$

$$\mathbf{P}_{\xi,j+1}^- = \mathbf{\Phi}_{\xi,j} \mathbf{P}_{\xi,j}^+ \mathbf{\Phi}_{\xi,j} + \mathbf{Q}_{\xi,j}, \quad \mathbf{\Phi}_{\xi,j} = \begin{pmatrix} \mathbf{\Phi}_j & \mathbf{E} & 0 \\ 0 & \mathbf{\Gamma}_{I,j} & 0 \\ 0 & 0 & \mathbf{\Gamma}_{II,j} \end{pmatrix}, \ \mathbf{Q}_{\xi,j} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{\Theta}_{I,j} & 0 \\ 0 & 0 & \mathbf{\Theta}_{II,j} \end{pmatrix}, \ \mathbf{C}_j = \begin{pmatrix} \mathbf{E} - \mathbf{K}_j \mathbf{H}_j & 0 & -\mathbf{K}_j \\ 0 & \mathbf{E} & 0 \\ 0 & 0 & \mathbf{E} \end{pmatrix}$$

$$\mathbf{P}_{\xi,j}^+ = \mathbf{C}_j \mathbf{P}_{\xi,j}^- \mathbf{C}_j^T$$

EKF Adjustment Technique

- If the motion is close to quasi stationary then one can calculate the asymptotical value of error matrix $\mathbf{P}_{\xi,\infty}$

$$\mathbf{P}_{\xi,\infty} = \mathbf{C}_{\xi}\left[\mathbf{\Phi}_{\xi}\mathbf{P}_{\xi,\infty}\mathbf{\Phi}_{\xi}^{T} + \mathbf{Q}_{\xi}\right]\mathbf{C}_{\xi}^{T}.$$

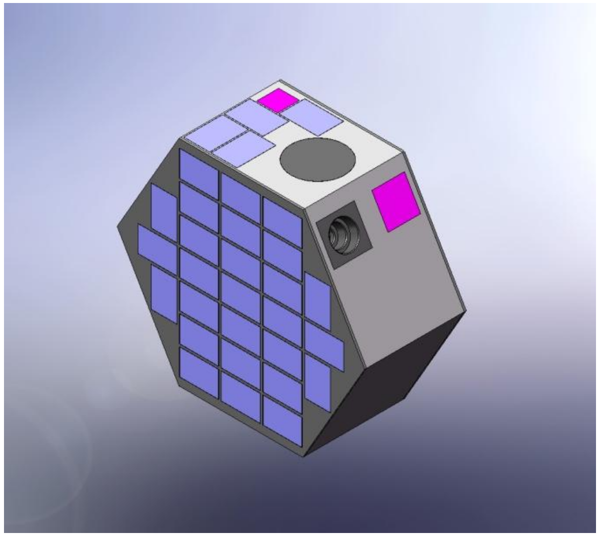
- Filter adjustment problem

$$\{\mathbf{Q},\ \mathbf{R}\} = \arg\min(\mathrm{tr}\,\mathbf{P}_{\xi,\infty}).$$

- The problem can be solved by gradient descent method calculating the accuracy in certain parameters area with defined step.

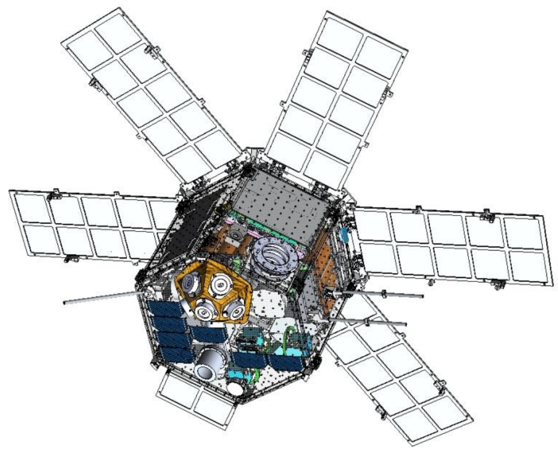
TabletSat microsatellite series

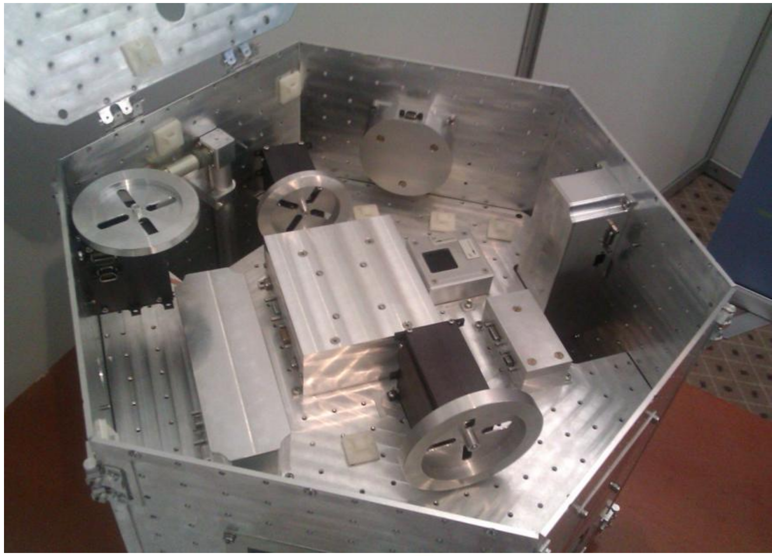
- Form is hexagonal prism (1 Unit)
- 1U module weight is about 10 kg
- Modular principle of microsatellite building
- The microsatellite consists of unified elements
- Plug-n-Play Architecture, describing mechanical, electrical and data interfaces
- Depending on the mission requirements a specific set of ADCS actuators and sensors are installed on board TabletSat.



TabletSat-Aurora microsatellite

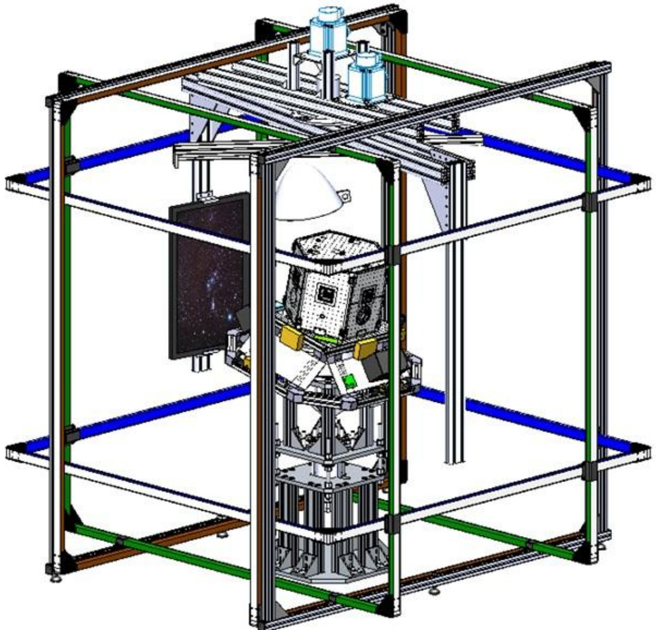
- Micro-satellite technology demonstrator
- Planning launch is in the summer of 2014
- Mass is about 25kg.
- Three-axis ADCS:
    - Flywheels
    - Gimbals
    - Star sensor
    - Sun sensors
    - Magnetometer
    - Gyro





Attitude determination sensors parameters

Parameter	Magnetometer	Sun sensor	Gyro	Star sensor
Instrument range	± 200 000 nT	± 60 deg	± 250 deg/s	–
Standard error	250 nT	0.1 deg	0.005 deg/s	0.001 deg



An Example: Kalman Filter Based on Magnetometer and Sun Sensor Measurements

State vector: $\mathbf{x}(t)=[\mathbf{q}(t)\ \boldsymbol{\omega}(t)]^T$

Attitude motion equations: $\mathbf{J}\dot{\boldsymbol{\omega}}_{si}=\mathbf{N}_{ctrl}+\mathbf{N}_{gg}+\mathbf{N}_{dist}-\boldsymbol{\omega}_{si}\times\left(\mathbf{J}\boldsymbol{\omega}_{si}+\boldsymbol{h}\right)$

$$\dot{\mathbf{q}}_{so}=\frac{1}{2}\mathbf{q}_{so}\circ\boldsymbol{\omega}_{so},$$

Linearized motion equations:

$$\delta\dot{\mathbf{x}}=\mathbf{F}\delta\mathbf{x},\ \mathbf{F}(t)=\begin{bmatrix}-W_{\omega} & \frac{1}{2}\mathbf{E}_{3\times3}\\ \mathbf{J}^{-1}\left(k\mathbf{F}_g-\mathbf{K}_{\alpha}\mathbf{W}_{\Lambda_{rel}}\right)_{3\times3} & -\mathbf{J}^{-1}K_{\omega}\end{bmatrix}_{6\times6}$$

Measurement model:

$$\mathbf{z}=\begin{pmatrix}\mathbf{b}_{meas}\\ \mathbf{s}_{meas}\end{pmatrix}=\begin{pmatrix}A_{so}(\mathbf{q})\mathbf{b}_o\\ A_{so}(\mathbf{q})\mathbf{s}_o\end{pmatrix}+\mathbf{v}$$

Linearized measurement model:

$$\delta\mathbf{z}=\mathbf{H}\delta\mathbf{x},\quad \mathbf{H}=\begin{bmatrix}h_1\mathbf{b}_o & h_2\mathbf{b}_o & h_3\mathbf{b}_o & 0_{3\times3}\\ h_1\mathbf{s}_o & h_2\mathbf{s}_o & h_3\mathbf{s}_o & 0_{3\times3}\end{bmatrix}_{6\times6},\quad h_i=\left[\frac{\partial A(q_k)}{\partial q_{k,i}}\right]$$

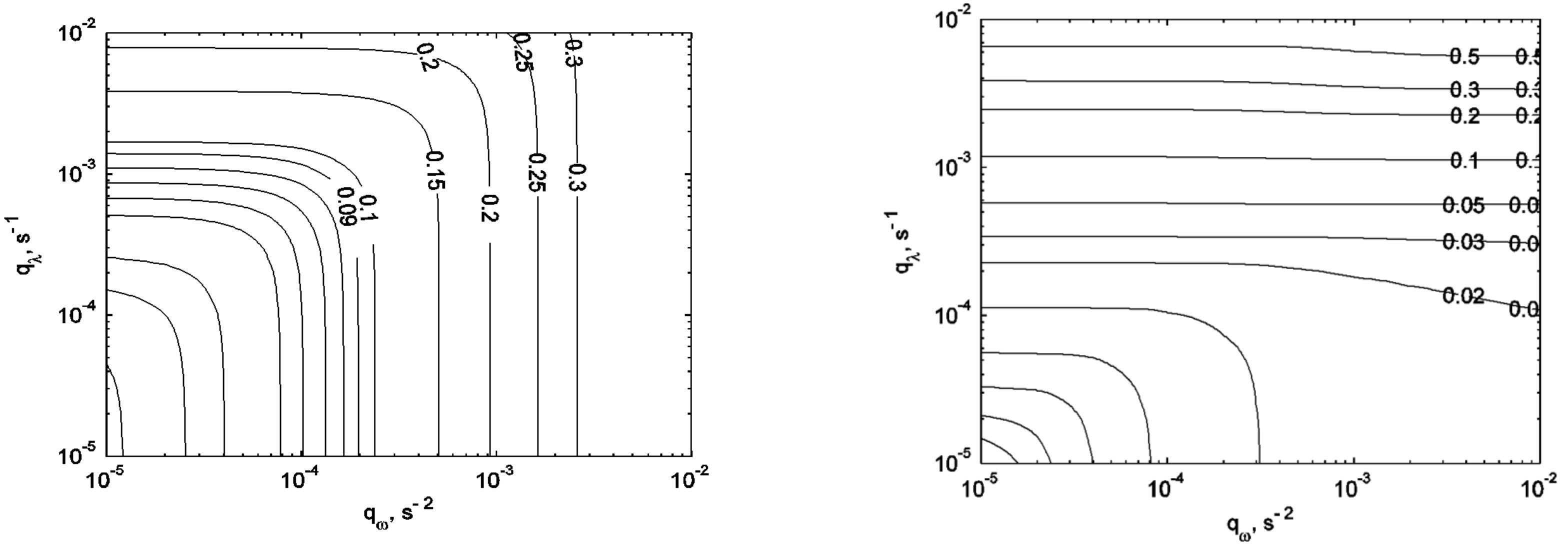
Accuracy investigation for quasi stationary motion

- Assume the error matrix $\mathbf{R}$ is known the dynamic error matrix is

$$Q = diag\left(q_{\lambda}^{2},\, q_{\lambda}^{2},\, q_{\lambda}^{2},\, q_{\omega}^{2},\, q_{\omega}^{2},\, q_{\omega}^{2}\right)$$

- To estimate the filter accuracy calculate P_{∞}, the errors are

$$\sigma_{\lambda} = \sqrt{\max\left(p_{11}, p_{22}, p_{33}\right)},\ \sigma_{\omega} = \sqrt{\max\left(p_{44}, p_{55}, p_{66}\right)}$$



Attitude (left) and angular velocity (right) determination accuracy dependence on parameters. Contours correspond to Euler angles accuracy levels in degree (left) and to angular velocity accuracy in degree per second (right).

Accuracy dependence on perturbation unaccounted in motion equation

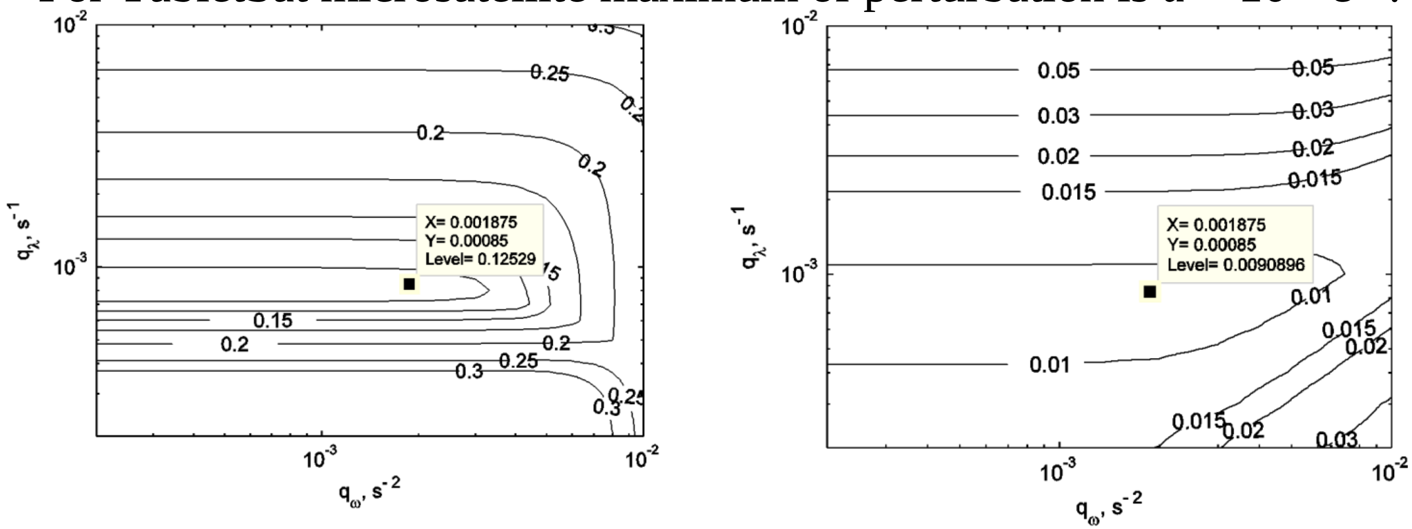
- Assume the real equation of motion

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + \boldsymbol{\chi}_k, \quad \boldsymbol{\chi}_{k+1} = E\boldsymbol{\chi}_k + \boldsymbol{\theta}_k,$$

- Consider the perturbation vector $\boldsymbol{\chi}$ to be

$$\boldsymbol{\chi} = \left[d\Delta t^2/2 \;\; d\Delta t^2/2 \;\; d\Delta t^2/2 \;\; d\Delta t \;\; d\Delta t \;\; d\Delta t \right]^T.$$

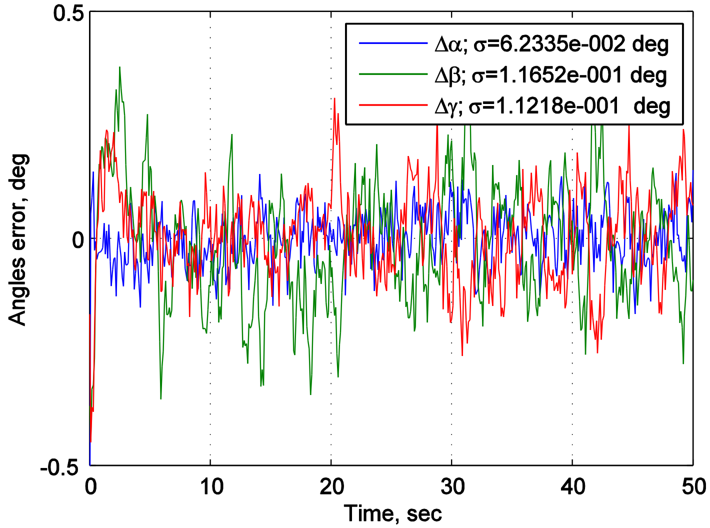
- For TabletSat microsatellite maximum of perturbation is $d = 10^{-6}\ \mathrm{c}^{-2}$.

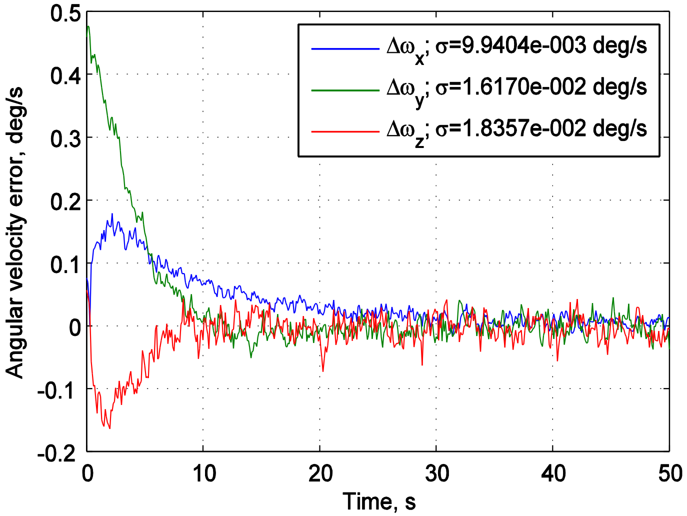


Attitude (left) and angular velocity (right) determination accuracy dependence on parameters under and constant perturbation torque . Contours correspond to Euler angles accuracy levels in degree (left) and to angular velocity accuracy in degree per second (right).

Comparison with EKF simulation results

- The maximum of accuracy achieves at $q_\lambda = 8 \cdot 10^{-4}\ c^{-1}$ and $q_\omega = 2 \cdot 10^{-3}\ c^{-2}$.
- The maximum of accuracy $\sigma_\varphi = 0.12$ deg and $\sigma_\omega = 0.009$ deg/ s
- These values are in a good correspondence with EKF simulation data.

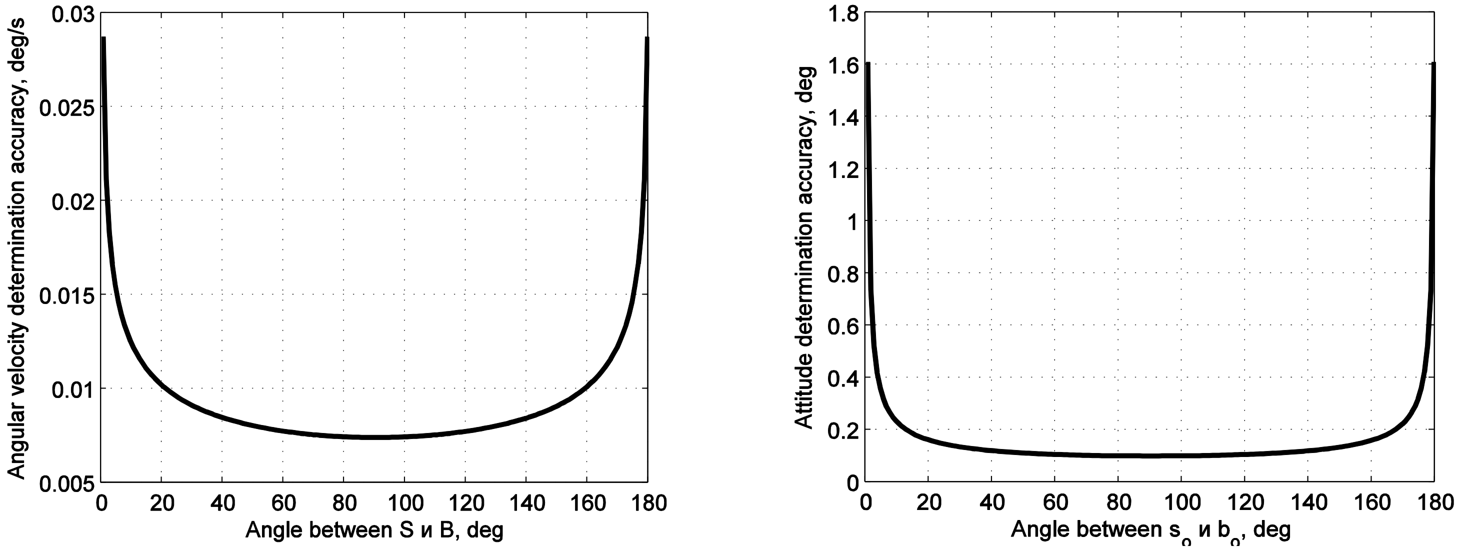




Attitude (a) and angular velocity (b) errors during Kalman filter work simulation with parameters $q_\lambda = 8 \cdot 10^{-4}\ c^{-1}$, $q_\omega = 2 \cdot 10^{-3}\ c^{-2}$ **, and constant perturbation torque**

Comparison with EKF simulation results

- The accuracy decreases when angle between magnetic field vector $\mathbf{b}_0$ and sun direction vector $\mathbf{s}_0$ differs from 90 degrees.
- If during the satellite exploitation the angle between $\mathbf{b}_0$ and $\mathbf{s}_0$ is less than 10 degree or more then 170 degree the accuracy of Euler anglesestimation is unsatisfactory.



The best attitude (left) and angular velocity (right) determination accuracy dependence on angle between magnetic field direction and sun direction under constant perturbation torque

Conclusions

- The proposed method of Kalman filter performance study is an effective instrument for accuracy analysis and filter tuning in quasi-stationary satellite motion case.
- The approach allows to estimate the influence of unaccounted perturbation on a motion determination accuracy.
- The proposed advanced method for Kalman filter performance adjustment and study is applied for a set of the algorithms of "TabletSat" microsatellites series.

Acknowledgments

- This work was supported by the Russian Foundation for Basic Research (Grant no. 14-01-31313, 13-01-00665, 12-01-33045) and the RF Ministry of Science and Education
- This work was performed in the framework of the contract with “SputniX” Ltd (contract № 1226\11-1)

Thanks for your attention!