Advanced Technique for Kalman Filter Adjustment and Its Implementation Onboard of "TabletSat" Microsatellite Series

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- Kalman Filter Adjustment Technique
- TabletSat Details
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Algorithms based on Kalman filter

The satellite motion model
\[ \dot{x} = f(x, t) + q_k, \]
\[ M(q_k) = 0, M(q_k q_k^T) = Q_k, \]

The measurement model
\[ z_k = h(x_k, t) + r_k. \]
\[ M(r_k) = 0, M(r_k r_k^T) = R_k. \]

Linearization
\[ F_k = \frac{\partial f(x, t)}{\partial x} \bigg|_{x = \hat{x}_{k-1}} \]
\[ H_k = \frac{\partial h(x, t)}{\partial x} \bigg|_{x = \hat{x}_{k-1}} \]
\[ \Phi_k = E + F_k (t_k - t_{k-1}). \]

Prediction stage
1. Project the state ahead
\[ \hat{x}^-_k = \hat{x}^+_k + \int_{t_{k-1}}^{t_k} f(x, t) dt, \]
2. Project the error covariance ahead
\[ P^-_k = \Phi_k P^+_k \Phi_k^T + Q_k, \]

Correction stage
1. Compute the Kalman Gain
\[ K_k = P^-_k H_k^T (H_k P^-_k H_k^T + R_k)^{-1}, \]
2. Update the estimate via \( z_k \)
\[ \hat{x}^+_k = \hat{x}^-_k + K_k [z_k - h(\hat{x}^-_k, t)], \]
3. Update the error covariance
\[ P^+_k = [E - K_k H_k] P^-_k. \]
The problem of EKF adjustment

- The real dynamic and measurement models differ from the ones used in EKF.
- Then the dynamical model error matrix Q and measurement model error matrix R become the adjustment parameters.
- The problem is to choose Q and R to increase the EKF estimation accuracy.
## Kalman Filter Adjustment Techniques

<table>
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<th>Investigation technique</th>
<th>Shortcomings</th>
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<tr>
<td>Monte-Karlo simulations (Mortari D., Markley L.S., Steyn W.H., Matthews O., Maybeck P.)</td>
<td>Time-consuming</td>
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<td>Genetic algorithms (Bar-Itzhack, Oshman Y., Clements R.)</td>
<td>Time-consuming, obtain local minimum only</td>
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<td>Adaptive Filtering (Bekir E., Ng A., Kumar K.)</td>
<td>Require more computational power</td>
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<td>Covariance matrix computation for stationary system (Parusnikov N., Golovan A., Balacrishnan A.)</td>
<td>Applicable only for stationary system</td>
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We propose the advanced analytical adjustment technique for Kalman Filter performance investigation
Accuracy investigation for quasi stationary motion

For stationary system:
\( \Phi_k = \Phi = const, \ H_k = H = const, \ Q_k = Q = const, \ R_k = R = const, \)
the asymptotic error matrix \( P_\infty \) after convergence is existing and can be obtained from equation

\[
P_\infty = \left\{ E - (\Phi P_\infty \Phi^T + Q)H^T \left[ H(\Phi P_\infty \Phi^T + Q)H^T + R \right]^{-1} H \right\}(\Phi P_\infty \Phi^T + Q).
\]

Assumptions:
- The motion is slow, matrices \( \Phi, H \) are close to be constant during sample time.
- The EKF converge at the particular matrices \( \Phi \) and \( H \), and one can calculate \( P_\infty \).
- The diagonal elements of \( P_\infty \) is an accuracy estimation.
Investigation of Influence of Unaccounted in EKF Perturbations

<table>
<thead>
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<th>Kalman filter equations</th>
<th>&quot;Truth model&quot; equations</th>
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<tr>
<td>Dynamical model</td>
<td>$x_{k+1} = \Phi_k x_k + q_k, \ M(q_k q_k^T) = Q_k$</td>
<td>$x_{k+1} = \Phi_k x_k + y_{I,k}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y_{I,k+1} = \Gamma_{I,k} y_k + \theta_{I,k}$,</td>
</tr>
<tr>
<td>Measurement model</td>
<td>$z_k = H_k x_k + r_k, \ M(r_k r_k^T) = R_k$</td>
<td>$z_k = H_k x_k + y_{II,k}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y_{II,k+1} = \Gamma_{II,k} y_{II,k} + \theta_{II,k}$</td>
</tr>
<tr>
<td>Prediction stage</td>
<td>$\hat{x}^-_{k+1} = \Phi_k \hat{x}_k^+$</td>
<td>$\hat{x}^+<em>{k} = (E - K_k H_k) \hat{x}^-</em>{k} - K_k y_{II,k}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y^+<em>{I,k} = y^-</em>{I,k}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y^+<em>{II,k} = y^-</em>{II,k}$</td>
</tr>
<tr>
<td>Correction stage</td>
<td>$\hat{x}^+<em>{k} = \hat{x}^-</em>{k} + K_k (z_k - H_k \hat{x}^-_{k})$</td>
<td>$x_{k+1} = \Phi_k x_k + y_{I,k}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y_{I,k+1} = \Gamma_{I,k} y_{I,k} + \theta_{I,k}$,</td>
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<td>$y_{II,k+1} = \Gamma_{II,k} y_{II,k} + \theta_{II,k}$</td>
</tr>
</tbody>
</table>

Calculation of the error matrix $P$ for the extended state vector:

$$\xi = (x, y_I, y_{II})^T,$$

$P_{\xi,j} = \Phi_{\xi,j} P_{\xi,j}^+ \Phi_{\xi,j}^T + Q_{\xi,j}$,  $\Phi_{\xi,j} = \begin{pmatrix} \Phi_j & E & 0 \\ 0 & \Gamma_{I,j} & 0 \\ 0 & 0 & \Gamma_{II,j} \end{pmatrix}$,  $Q_{\xi,j} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Theta_{I,j} & 0 \\ 0 & 0 & \Theta_{II,j} \end{pmatrix}$,  $C_j = \begin{pmatrix} E - K_j H_j & 0 & -K_j \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$

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EKF Adjustment Technique

- If the motion is close to quasi stationary then one can calculate the asymptotical value of error matrix $P_{\xi,\infty}$

$$P_{\xi,\infty} = C_{\xi} \left[ \Phi_{\xi} P_{\xi,\infty} \Phi_{\xi}^T + Q_{\xi} \right] C_{\xi}^T.$$ 

- Filter adjustment problem

$$\{Q, R\} = \arg \min(\text{tr } P_{\xi,\infty}).$$

- The problem can be solved by gradient descent method calculating the accuracy in certain parameters area with defined step.
TabletSat microsatellite series

- Form is hexagonal prism (1 Unit)
- 1U module weight is about 10 kg
- Modular principle of microsatellite building
- The microsatellite consists of unified elements
- Plug-n-Play Architecture, describing mechanical, electrical and data interfaces
- Depending on the mission requirements a specific set of ADCS actuators and sensors are installed on board TabletSat.
TabletSat-Aurora microsatellite

- Micro-satellite technology demonstrator
- Planning launch is in the summer of 2014
- Mass is about 25 kg.
- Three-axis ADCS:
  - Flywheels
  - Gimbals
  - Star sensor
  - Sun sensors
  - Magnetometer
  - Gyro

Attitude determination sensors parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Magnetometer</th>
<th>Sun sensor</th>
<th>Gyro</th>
<th>Star sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument range</td>
<td>± 200 000 nT</td>
<td>± 60 deg</td>
<td>± 250 deg/s</td>
<td>–</td>
</tr>
<tr>
<td>Standard error</td>
<td>250 nT</td>
<td>0.1 deg</td>
<td>0.005 deg/s</td>
<td>0.001 deg</td>
</tr>
</tbody>
</table>

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An Example: Kalman Filter Based on Magnetometer and Sun Sensor Measurements

State vector:

\[ x(t) = [q(t) \ \omega(t)]^T \]

Attitude motion equations:

\[ \dot{\omega}_{si} = \mathbf{N}_{ctrl} + \mathbf{N}_{gg} + \mathbf{N}_{dist} - \omega_{si} \times (\mathbf{J}\omega_{si} + \mathbf{h}) \]

\[ \dot{q}_{so} = \frac{1}{2} q_{so} \times \omega_{so}, \]

Linearized motion equations:

\[ \delta \dot{x} = \mathbf{F} \delta x, \quad \mathbf{F}(t) = \begin{bmatrix} -W_{\omega} & \frac{1}{2} \mathbf{E}_{3 \times 3} \\ \mathbf{J}^{-1} \left( k\mathbf{F}_g - K_{\alpha} \mathbf{W}_{\Lambda_{rel}} \right)_{3 \times 3} & -\mathbf{J}^{-1} K_{\omega} \end{bmatrix}_{6 \times 6} \]

Measurement model:

\[ z = \begin{bmatrix} \mathbf{b}_{meas} \\ \mathbf{s}_{meas} \end{bmatrix} = \begin{bmatrix} A_{so}(q) \mathbf{b}_o \\ A_{so}(q) \mathbf{s}_o \end{bmatrix} + \mathbf{v} \]

Linearized measurement model:

\[ \delta z = \mathbf{H} \delta x, \quad \mathbf{H} = \begin{bmatrix} h_1 \mathbf{b}_o & h_2 \mathbf{b}_o & h_3 \mathbf{b}_o & 0_{3 \times 3} \\ h_1 \mathbf{s}_o & h_2 \mathbf{s}_o & h_3 \mathbf{s}_o & 0_{3 \times 3} \end{bmatrix}_{6 \times 6}, \quad h_i = \frac{\partial A(q_k)}{\partial q_{k,i}} \]
Accuracy investigation for quasi stationary motion

• Assume the error matrix $R$ is known the dynamic error matrix is

$$Q = \text{diag}\left( q_1^2, q_2^2, q_3^2, q_4^2, q_5^2, q_6^2 \right)$$

• To estimate the filter accuracy calculate $P_\infty$, the errors are

$$\sigma_\lambda = \sqrt{\max\left( p_{11}, p_{22}, p_{33} \right)}, \quad \sigma_\omega = \sqrt{\max\left( p_{44}, p_{55}, p_{66} \right)}$$

Attitude (left) and angular velocity (right) determination accuracy dependence on parameters. Contours correspond to Euler angles accuracy levels in degree (left) and to angular velocity accuracy in degree per second (right).
Accuracy dependence on perturbation unaccounted in motion equation

- Assume the real equation of motion

\[ x_{k+1} = \Phi_k x_k + \chi_k, \quad \chi_{k+1} = E \chi_k + \theta_k, \]

- Consider the perturbation vector \( \chi \) to be

\[ \chi = \begin{bmatrix} d \Delta t^2 / 2 & d \Delta t^2 / 2 & d \Delta t^2 / 2 & d \Delta t & d \Delta t & d \Delta t \end{bmatrix}^T. \]

- For TabletSat microsatellite maximum of perturbation is \( d = 10^{-6} \text{ c}^{-2}. \)

Attitude (left) and angular velocity (right) determination accuracy dependence on parameters under and constant perturbation torque. Contours correspond to Euler angles accuracy levels in degree (left) and to angular velocity accuracy in degree per second (right).
Comparison with EKF simulation results

- The maximum of accuracy achieves at $q_\alpha = 8 \cdot 10^{-4} \, \text{c}^{-1}$ and $q_\omega = 2 \cdot 10^{-3} \, \text{c}^{-2}$.
- The maximum of accuracy $\sigma_\phi = 0.12 \, \text{deg}$ and $\sigma_\omega = 0.009 \, \text{deg/s}$
- These values are in a good correspondence with EKF simulation data.

Attitude (a) and angular velocity (b) errors during Kalman filter work simulation with parameters $q_\alpha = 8 \cdot 10^{-4} \, \text{c}^{-1}$, $q_\omega = 2 \cdot 10^{-3} \, \text{c}^{-2}$, and constant perturbation torque
Comparison with EKF simulation results

- The accuracy decreases when angle between magnetic field vector $b_0$ and sun direction vector $s_0$ differs from 90 degrees.
- If during the satellite exploitation the angle between $b_0$ and $s_0$ is less than 10 degree or more than 170 degree the accuracy of Euler angles estimation is unsatisfactory.

The best attitude (left) and angular velocity (right) determination accuracy dependence on angle between magnetic field direction and sun direction under constant perturbation torque
Conclusions

- The proposed method of Kalman filter performance study is an effective instrument for accuracy analysis and filter tuning in quasi-stationary satellite motion case.

- The approach allows to estimate the influence of unaccounted perturbation on a motion determination accuracy.

- The proposed advanced method for Kalman filter performance adjustment and study is applied for a set of the algorithms of "TabletSat" microsatellites series.
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Thanks for your attention!