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Geometrical Tools for the Systematic Design of Low-Energy Transfers in the Earth-Moon-Sun System

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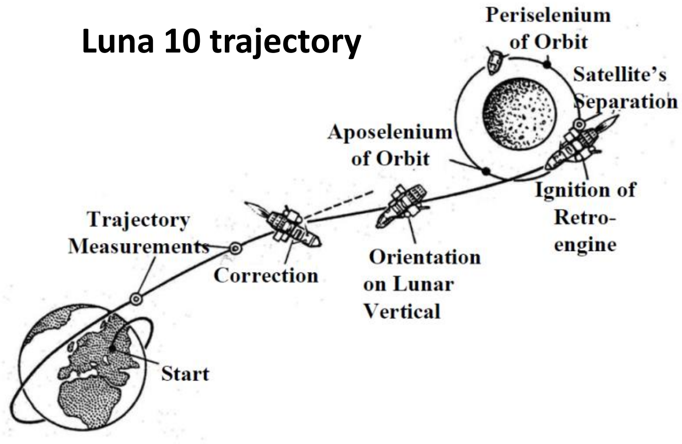
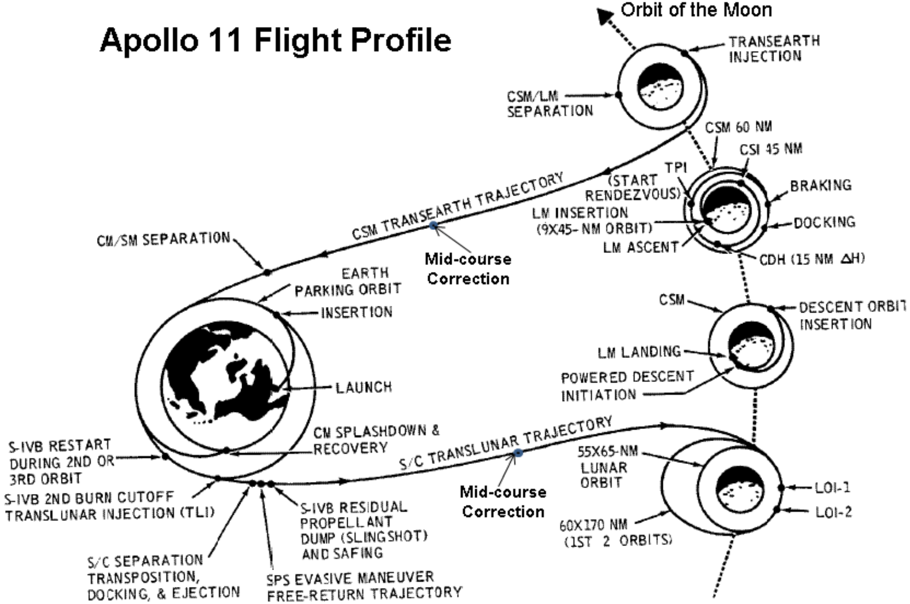
From patched conic approximation…

Luna 10 trajectory



Credit: V.V. Ivashkin. Lunar trajectories of the spacecraft . – 2008.

Luna 10 (1966) is the first artificial satellite of the Moon



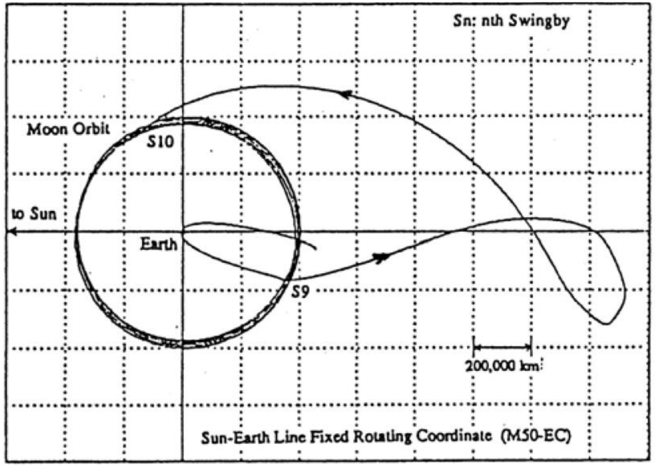
To get to a lunar orbit, large space probes (e.g., Apollo 11) have to perform a high ΔV lunar orbit insertion (LOI) maneuver

Credit: https://www.mpoweruk.com/Apollo_Moon_Shot.htm

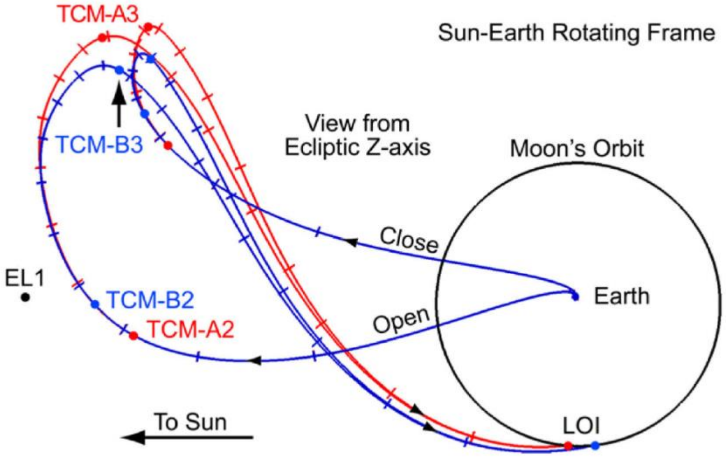
…to low-energy WSB transfers

➢ Compared to the high-energy transfers:

- the lower cost
- the enlarged launch windows
- the extended transfer time



Hiten (1991) trajectory

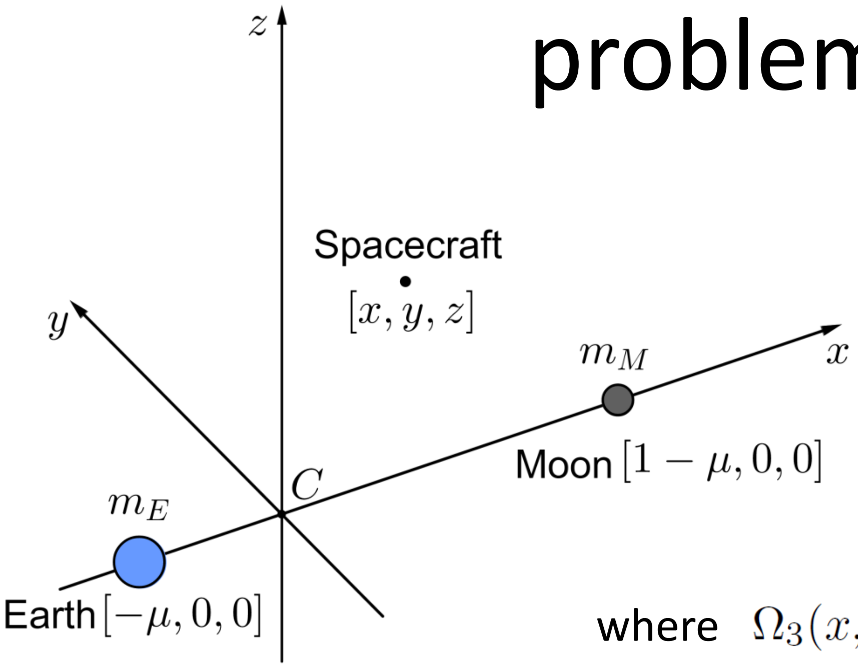


GRAIL (2011) lunar transfer

Credit: Anderson R. L., Parker J. S. Targeting low-energy transfers to low lunar orbit. – 2011

Credit: Nishimura T., Kawaguchi J. On the Guidance and Navigation of Japanese Spacecraft" HITEN“. – 1993

Circular restricted three-body problem (CR3BP)



Equations of motion:

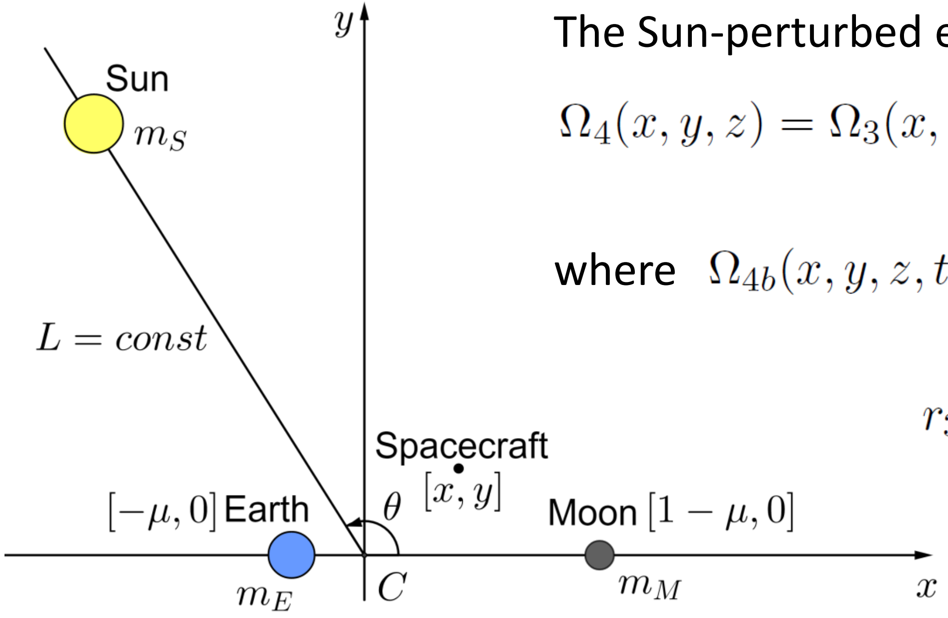
$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega_3}{\partial x},$$
$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega_3}{\partial y},$$
$$\ddot{z} = \frac{\partial \Omega_3}{\partial z}$$

where $\Omega_3(x, y, z) = \dfrac{x^2 + y^2}{2} + \dfrac{1-\mu}{r_1} + \dfrac{\mu}{r_2} + \dfrac{\mu(1-\mu)}{2},$

$\mu = m_M/(m_E + m_M)$; r_1, r_2 are the distances from the s/c to the Earth and the Moon

The Jacobi integral: $\boxed{J(x, y, z, \dot{x}, \dot{y}, \dot{z}) = 2\Omega_3(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)}$

Bicircular restricted four-body problem (BR4BP)



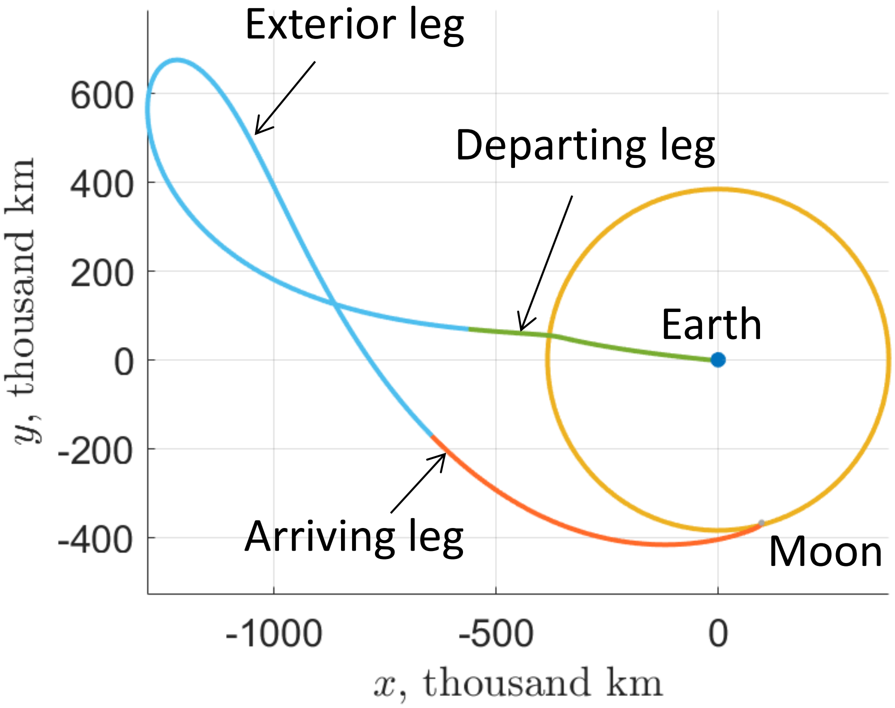
The Sun-perturbed effective potential:

$$\Omega_4(x, y, z) = \Omega_3(x, y, z) + \Omega_{4b}(x, y, z, t)$$

where $\Omega_{4b}(x, y, z, t) = \dfrac{Gm_S}{r_3(t)} - \dfrac{Gm_S}{L^2}(x\cos\theta(t) + y\sin\theta(t)),$

$r_3(t)$ is the distance from the s/c to the Sun

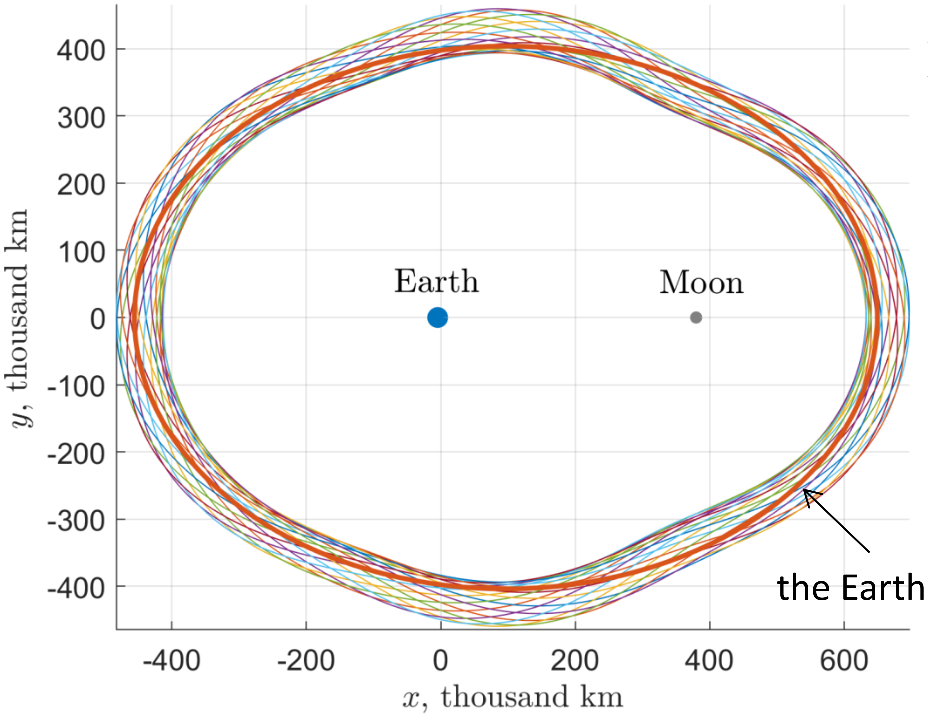
Structure of WSB trajectories



Example of WSB trajectory

- ➢ *Departing and arriving legs:* the Earth-Moon CR3BP
- ➢ *Exterior leg:* the Earth-Moon-Sun BR4BP

Earth-Moon region of prevalence

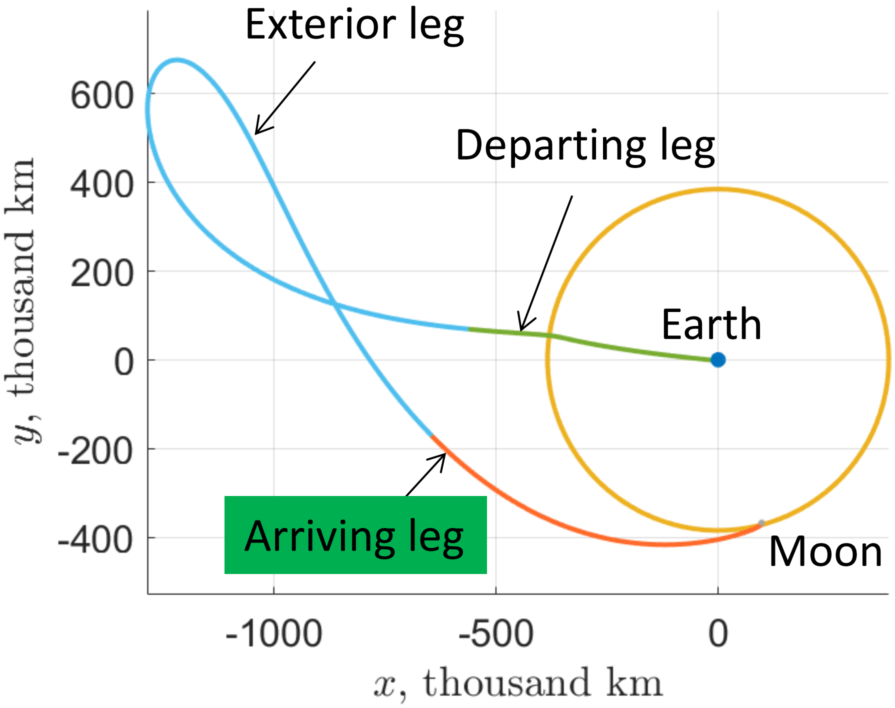


- The boundary of *the Earth-Moon region of prevalence* * : points in the configuration space where the error in the right-hand side of the spacecraft's equations of motion have the same magnitude independently of what body we neglect in the Earth-Moon-Sun system — the Moon or the Sun

the Earth-Moon mean-square averaged region of prevalence

* R. Castelli, "Regions of Prevalence in the Coupled Restricted Three-Body Problems Approximation," Communications in Nonlinear Science and Numerical Simulation, Vol. 17, No. 2, 2012, pp. 804–816.

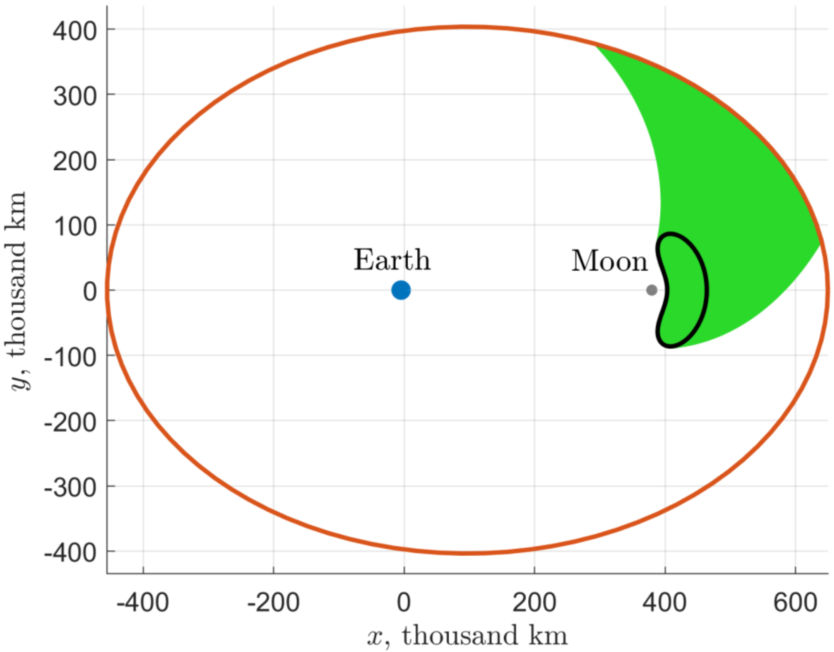
Structure of WSB trajectories



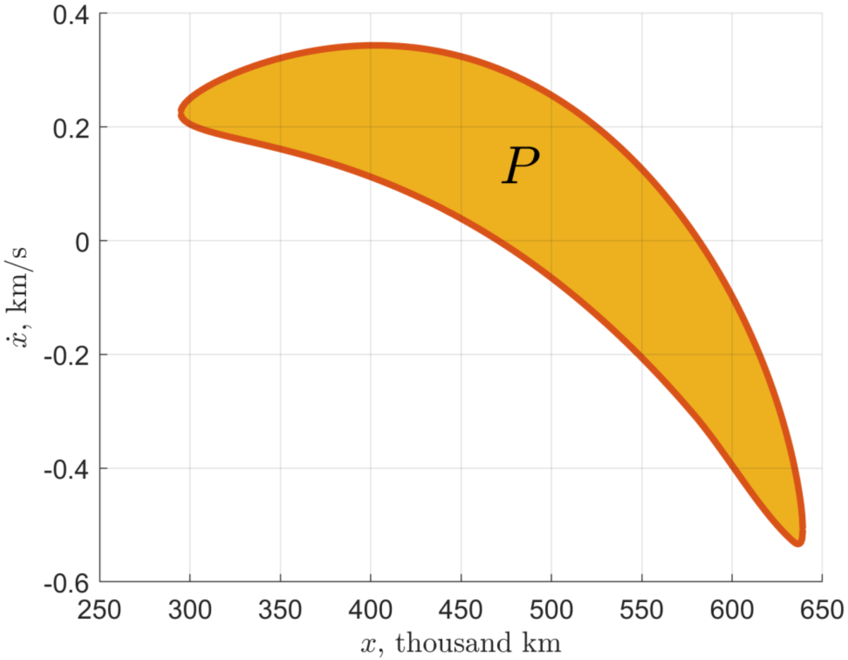
Example of WSB trajectory

- *Departing and arriving legs:* the Earth-Moon CR3BP
- *Exterior leg:* the Earth-Moon-Sun BR4BP

Lunar transit trajectories



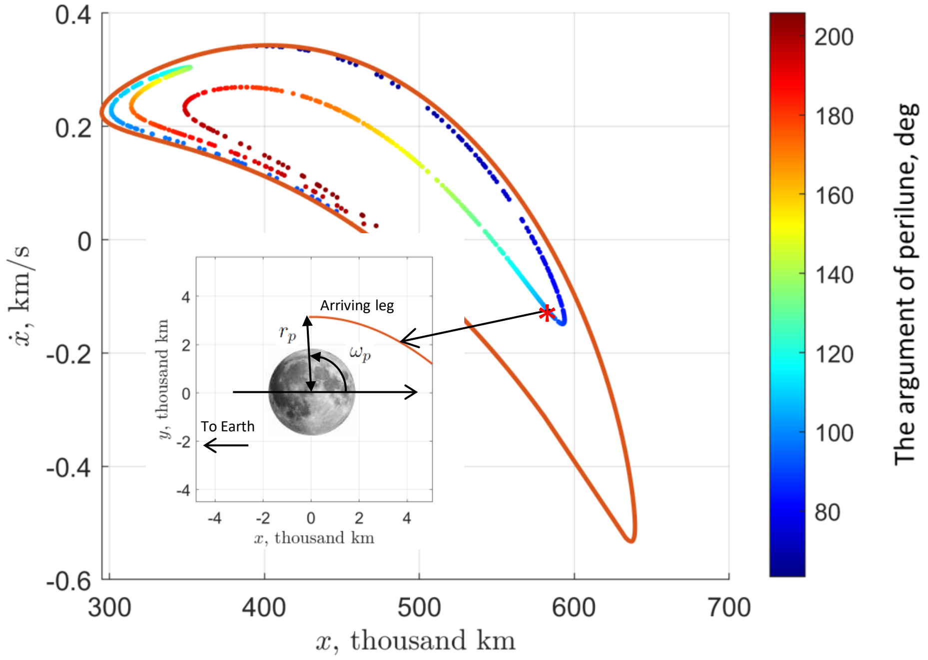
The stable manifold of the $J_{EM} = 3.06$ planar Lyapunov orbit



Lunar L_2 gateway P on the $(x, \dot{x})$ plane

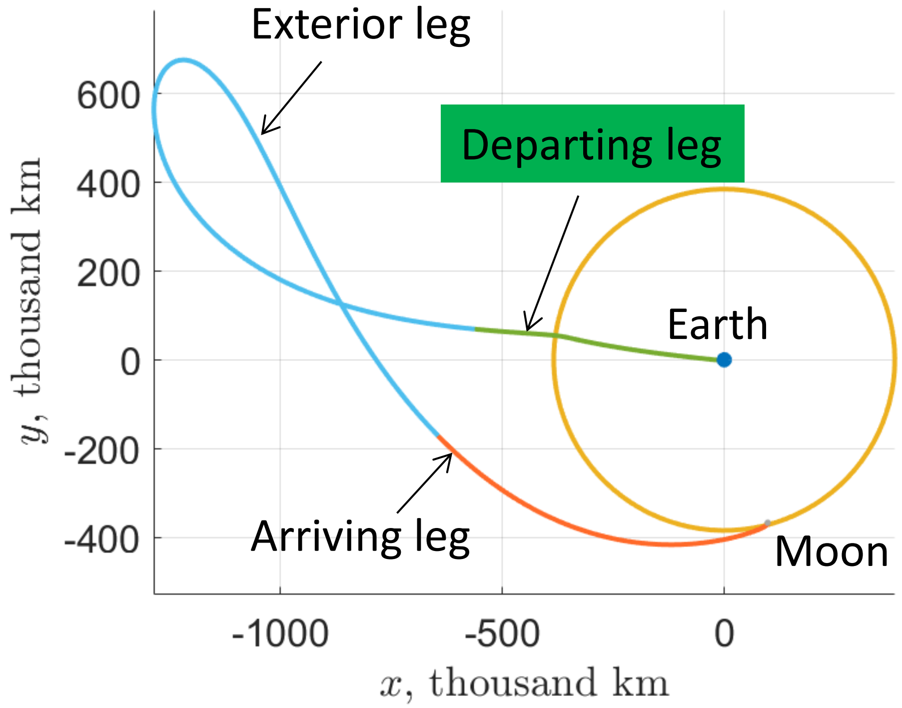
Synthesis of arriving legs

- For any point of P and a given $[x_P, \dot{x}_P]$, $\dot{y}_P$ is determined by $J(x_P, y_P, \dot{x}_P, \dot{y}_P) = J_{EM}$, y_P belongs to the region of prevalence boundary
- P collapses to a point when $J_{EM} \approx 3.18$
- The required LOI impulse at the perilune is estimated from $\Delta J_{EM} \approx \Delta v^2 + 2v\Delta v$



The perilune altitude contour line corresponding to the NRHO 9:2 perilune altitude 1403 km

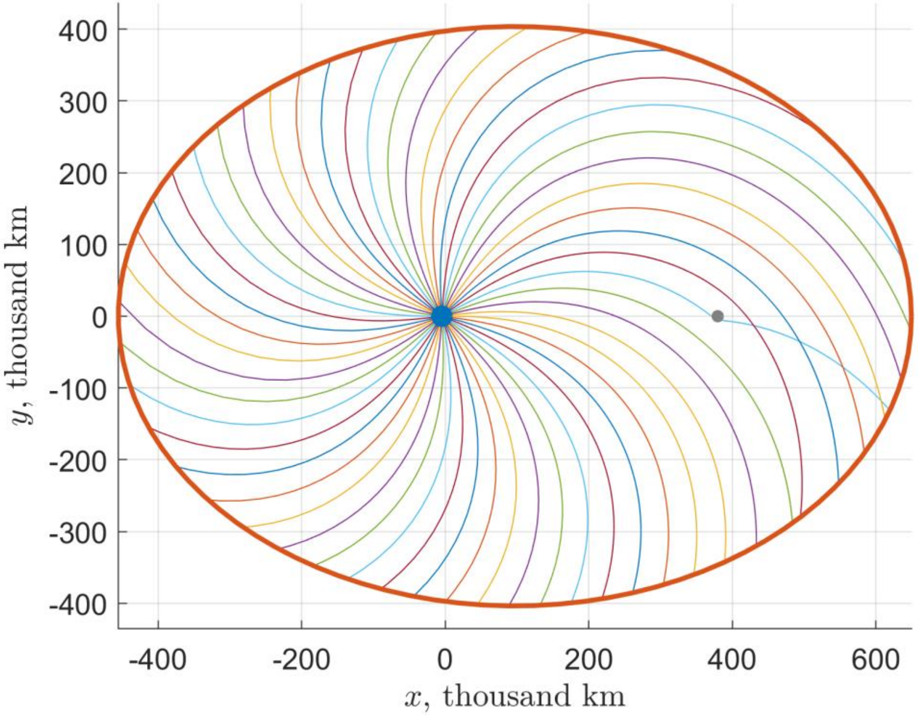
Structure of WSB trajectories



Example of WSB trajectory

- *Departing and arriving legs:* the Earth-Moon CR3BP
- *Exterior leg:* the Earth-Moon-Sun BR4BP

Earth collision trajectories



Earth collision trajectories with $J_{EM} = 3.06$

- The Levi-Chivita transformation

  $x + \mu + iy = (u + iv)^2, i^2 = -1$

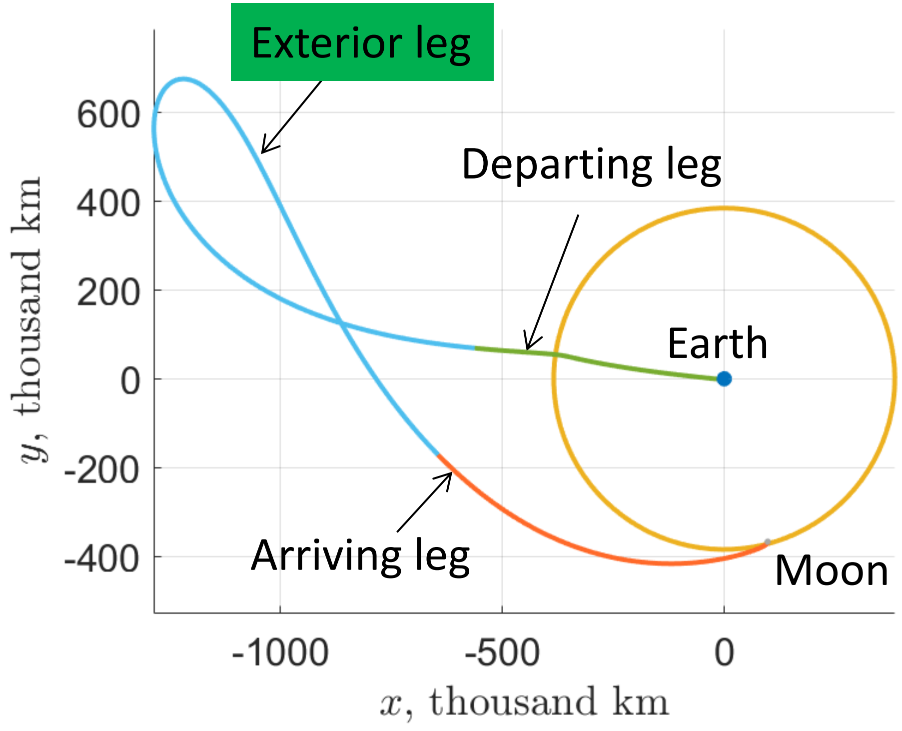
  $dt = r d\tau$

- The equations of motion in new variables

$$u'' = \frac{f_1(u,v)}{4} + 2(u^2 + v^2)v',$$
$$v'' = \frac{f_2(u,v)}{4} - 2(u^2 + v^2)u'$$

- Each collision trajectory depends on only two parameters: an ejection angle φ and a Jacobi constant J_{EM}

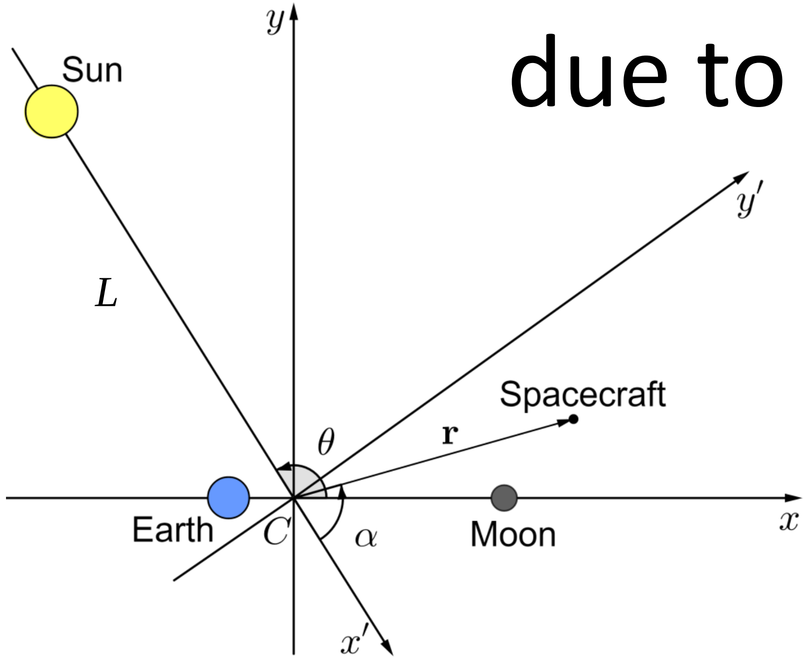
Structure of WSB trajectories



Example of WSB trajectory

- *Departing and arriving legs:* the Earth-Moon CR3BP
- *Exterior leg:* the Earth-Moon-Sun BR4BP

Jacobi integral change due to solar gravity



$$J_{EM} \simeq 3(1-\mu) + 2W_Z - 2E_2$$

E_2, W_Z the spacecraft Keplerian energy and the z-component of the orbital moment with respect to the Moon

$$\Delta E_2 \to \min \Rightarrow \Delta J_{EM} \to \max \Rightarrow \Delta J_{EM} > 0$$

$$\Delta J_{EM} = -2\Delta\Omega_{4b} + 2\int \frac{\partial \Omega_{4b}}{\partial \theta}\omega_s dt\ ,\ \omega_s = \frac{d\theta}{dt} < 0$$

$r/L << 1:$

$$\Omega_{4b} \approx \frac{Gm_S}{L} - \frac{1}{2}\frac{Gm_S}{L^3}r^2(1 - 3\cos^2\alpha)$$

$$\frac{\partial \Omega_{4b}}{\partial \theta} \approx \frac{3}{2}\frac{Gm_S}{L^3}r^2 \sin 2\alpha,$$

$\Rightarrow$ $\sin 2\alpha < 0$ when $r = r_{max}$

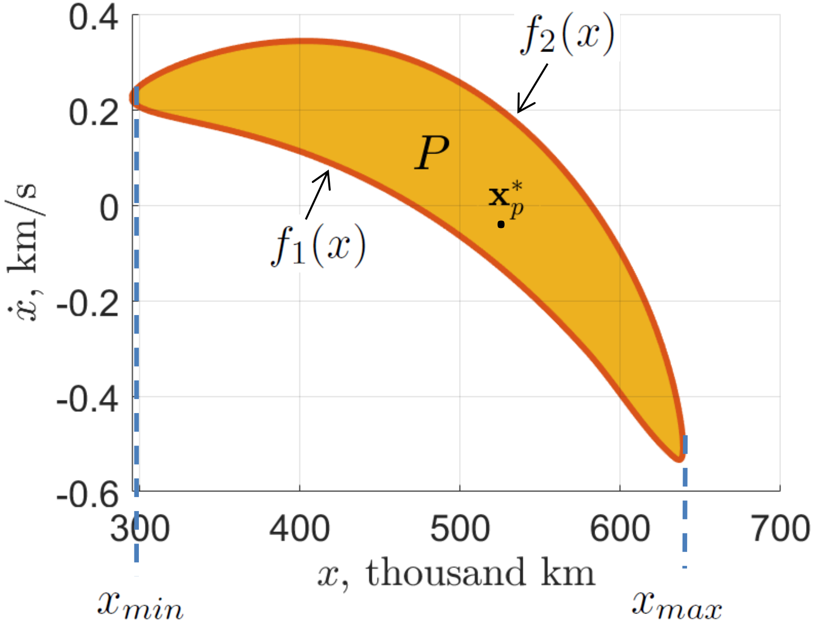
Designing planar WSB transfers

- *The optimization variables:* $J_{EM}^0, \varphi, \theta_0$
- *At the boundary point* $\mathbf{x}_p = [x_p, y_p, \dot{x}_p, \dot{y}_p]$:

$$f_1(x_p) \le \dot{x}_p \le f_2(x_p), \;\; x_{min} \le x_p \le x_{max}, \;\; y_p > 0,$$

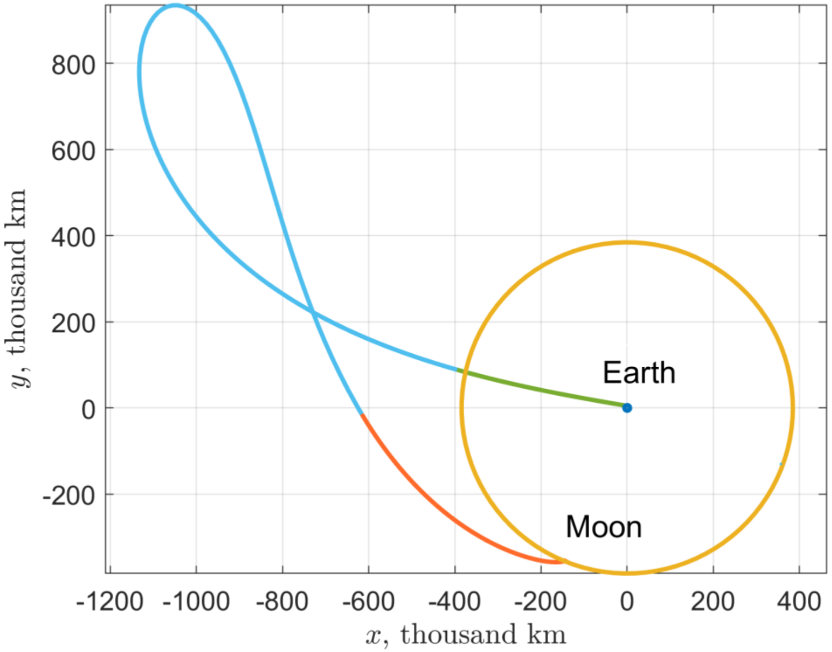
$$J_{EM}(\mathbf{x}_p) = J_{EM}^f,$$
$$t_p = t^* \text{ (if required)},$$
$$x_p = x_p^*,$$
$$\dot{x}_p = \dot{x}_p^*$$

- The apogee of the trajectory should lie in the II or IV quadrant of the $Cx'y'$ coordinate system

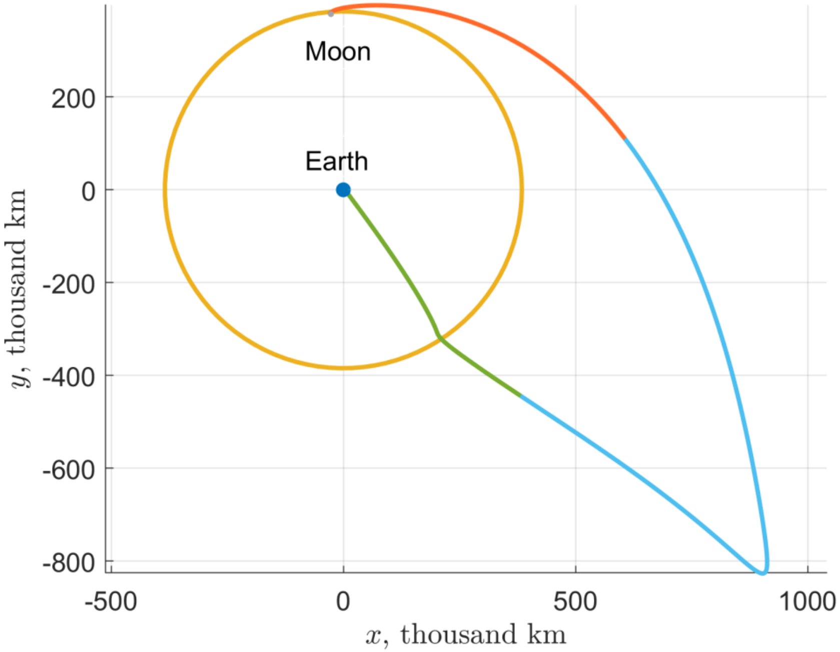


The gateway P corresponding to the desired value of the Jacoby integral J_{EM}^f

Examples of planar WSB trajectories



Planar WSB trajectory with $J_{EM}^{f} = 3.06$, $r_p = 3141$ km, $\omega_p = 119^\circ$, the time of flight is 87 days



Planar WSB trajectory with $J_{EM}^{f} = 3.06$, $r_p = 3141$ km, $\omega_p = 92^\circ$, the time of flight is 74 days

- *Optimization problem solver:* MATLAB's `fmincon` (the *sqp* option)

Adaptation to the ephemeris model

- *The high-fidelity model:* the central gravitational fields of the Earth and the Moon, gravitational perturbations from the Sun and all the planets of the Solar system, solar radiation pressure, GRGM1200A (8x8) harmonics for the lunar gravitational acceleration, JPL's DE430 ephemeris

- *Adaptation method:* multiple shooting

- *The optimization variables:*
  - the epochs and state vectors of the spacecraft,
  - lunar orbit injection (LOI) impulse,
  - trajectory correction maneuver (TCM)

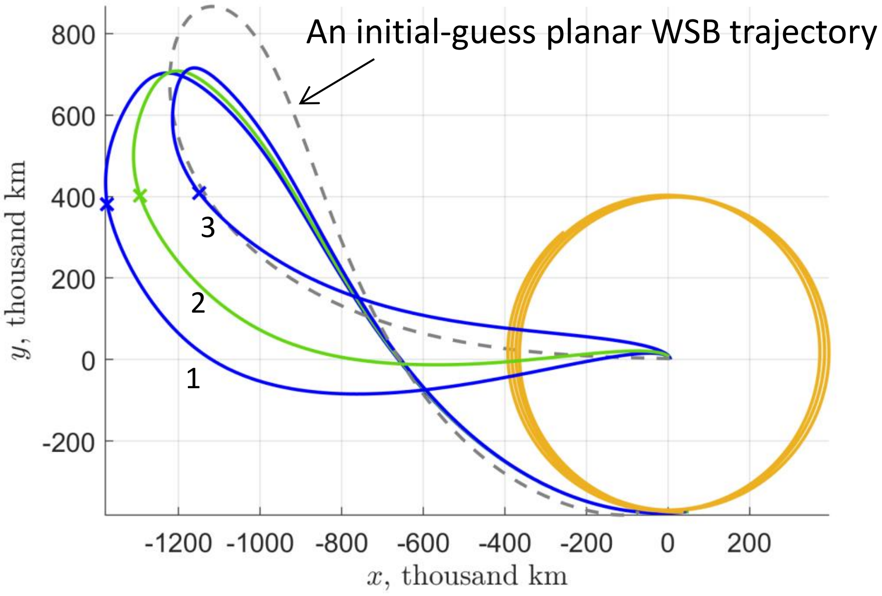
- *The objective function:*

$$\Delta V_{\sum} = \Delta V_{LOI}^2 + \Delta V_{TCM}^2 \rightarrow \min$$

- *The constraints* include requirements for
  - the altitude, inclination, and eccentricity of a post-launch parking near-Earth orbit,
  - the launch date and time,
  - the departure impulse magnitude (≤ 3.2 km/s),
  - smoothness of patching the position and velocity at all nodes,
  - conditions for entering the target orbit.

- *Launch window recovery:* continuation in the launch date

Realistic WSB trajectories



WSB trajectories from the orbit $h = 200$ km, $i = 51.6^{\circ}$ to the southern NRHO 9:2

The launch window is defined as $\Delta V_{\Sigma} \le 100$ m/s

1: the launch window opening,
$\Delta V_{\Sigma} = 32.876\,\text{m/s (TCM)} + 67.176\,\text{m/s (LOI)} = 100.052\,\text{m/s}$,
the start date is April 13, 2028, 12:00

2: the fuel-optimal transfer,
$\Delta V_{\Sigma} = 9.980\,\text{m/s (TCM)} + 66.734\,\text{m/s (LOI)} = 76.714\,\text{m/s}$,
the start date is April 20, 2028, 7:00

3: the launch window closing,
$\Delta V_{\Sigma} = 33.937\,\text{m/s (TCM)} + 66.096\,\text{m/s (LOI)} = 100.034\,\text{m/s}$,
the start date is April 28, 2028, 4:00

The arrival time is fixed: July 29, 2028, 08:13:29

- *Optimization problem solver:* MATLAB's `fmincon` (the *sqp* option)
- *Convergence from the initial guess* ~ 40 min; *a continuation step of 1 h in the start date* ≤ 10 s

Conclusion

- Planar initial-guess WSB trajectories corresponding to different launch dates and flight times have been successfully obtained in the BR4BP model of motion using geometrical and analytical tools presented in this study
- The adaptation of planar WSB trajectories to the realistic high fidelity model of motion was illustrated for the WSB transfer from the Baikonur launch parking orbit to the southern NRHO 9:2 for the launch date in April 2028. Convergence from the initial guess took no more than 40 minutes
- The family of WSB transfer trajectories for the whole launch window with the total cost $\Delta V_{\Sigma} \leq 100$ m/s was recovered by continuation in the launch date with a one-hour step. One step of the continuation method took approximately 10 s

The study is supported by the Russian Science Foundation (RSF) grant 19-11-00256